

# Stochastic resolution approach for production and remanufacturing decisions

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## Résumé

Cet article considère un système de production/retraitement à produit unique, multi-périodes avec une ligne hybride de fabrication de nouveaux produits et de retraitement des produits retournés. Compte tenu de la condition de “substitution parfaite”, nous formulons un problème linéaire MILP qui considère une demande et des retours déterministes ainsi qu’un modèle linéaire stochastique multi-étapes qui considère un processus stochastique pour la demande et les retours de produits en considérant que les décisions de lancement de lot sont optimales et ont déjà été prises. Nous proposons la méthode de programmation stochastique SDDP comme approche de résolution du problème formulé pour un horizon de planification finie. Ce travail est une extension de l’approche de la solution proposée dans (El Fassi et al 2014) où nous considérons un plan expérimental élargi pour la résolution stochastique. La méthode SDDP est appliquée pour des instances plus larges et des profils de demande différents. Les résultats sont rapportés dans la section des résultats préliminaires. Nous démontrons la performance et la robustesse de SDDP pour tous les scénarios échantillonnés..

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## Abstract -

This paper considers a multi-period single product production-inventory system with product returns and one hybrid manufacturing/remanufacturing line. Considering the “As good as new condition”, we formulate a MILP that considers deterministic demand and returns and a multi-stage stochastic linear problem MSSLP that considers stochastic data process for demand and returns while optimal lot decisions have already been taken. We propose the Stochastic Dual Dynamic Programming SDDP method as solution approach to solve the MSSLP over a finite planning horizon. This work is an extension of the solution approach proposed in (El Fassi and al 2014) where we consider a global experimental framework for stochastic resolution.

SDDP application for larger instances and different demand profiles is reported in the preliminary results section. We demonstrate the performance and robustness of SDDP over all sampled scenarios.

**Mots clés – Substitution parfaite, programmation stochastique, boucle d’approvisionnement fermée, retraitement**

**Keywords - Closed loop systems, lot sizing problem with returns, SDDP approach, remanufacturing, as good as new condition.**

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## 1 INTRODUCTION- STYLE PREMIER TITRE 1

In an environmental context, supply chain management should not be limited to « green » practices but should also include reverse logistics aspects and migrate to a closed loop system framework. Reverse flows refer to returns of end of life products for disposal or many reprocessing options including reuse, remanufacturing and recycling. (See Thierry et al 1995).

The consideration of these aspects affects necessarily the strategic and tactical/operational decision process.

The remanufacturing stakeholders are related to three different strategies that are available to every company that wants to join a reprocessing system. According to Lund (2003), reprocessing by original equipment manufacturer (OEM) takes place by providing the same production line. The OEM carries out the distribution process and sells to retailers or final consumers. The second option allows taking ownership of the remanufacturing operations by a conventional operator that can recover, reprocess

and sell directly to final consumers. Finally, the OEM may delegate this activity to a contractual operator.

We also underline the economic importance and size of remanufacturing industry (over than \$ 53 billion in the U.S.A. Lund (2003)). The study notes the lack of data for academic research and reveals many practical sides about the industry. The economic importance is supported by environmental issues that increase our interest in this research topic.

We are interested in this study by the OEM strategy which represents 6% of the entire industry. Hauser and Lund (2008) list 138 OEMs operating in different sectors in the United States. Many companies among them combine manufacturing and remanufacturing processes in the same hybrid lines. We refer the reader to Fuji -Film, Kodack and Fuji- Xerox in reprocessing of single-use cameras and photocopiers (Ostlin, Matsumoto and Umeda (2011), Hauser and Lund (2008)).

In the large literature review for quantitative models for inventory and production planning in closed loop systems, Akcalu and Centikya (2011), Junior and Filho (2011) recommend to involve uncertainty as a major aspect to ensure practical relevance. Uncertainty is related to timing and quantity of returns, quality of recovered products and also related to routing for remanufacturing operations. According to (Goodall and al 2014), remanufacturing operates in a variable level of uncertainty depending on remanufacturing decision process and remanufacturing stakeholder (OEM or independent remanufacturer). Thus a degree of risk is attributed with each decision made. Among the other relevant aspects, the authors recommend considering hybrid manufacturing/remanufacturing systems for OEMs, considering realistic costs and revenue structure encountered in practice and finally the interaction between demand and return process.

According to Junior and Filho (2011), 71.4% of the academic models developed to assess remanufacturing challenges are generic and not related to tactical/operational decisions (set of research questions conducted between 2000 and 2009). The first deterministic model involving returned items is introduced by Schrady (1967). Compared to a lot sizing problem, the lot sizing problem with returns LSPR is characterized by its NP-hard complexity for most of its variants (Helmrich et al., (2010)). This is justified by the addition of other decision aspects. Constraints mainly concern the remanufacturing capacity (in addition to the normal production), the balance flow at remanufacturing storage point and demand satisfaction by producing new products and reprocessed returns. For hybrid manufacturing/remanufacturing lines, the majority of studies are with separate/dedicated production and remanufacturing lines and rarely one hybrid line dedicated to two types of production lines (See Tang and Teunter (2006), Gungor and Gupta (1999), Lim (2011)).

In the literature, we can find many review studies which try to include uncertainty and propose stochastic variants of LSPR. However, to our knowledge, the stochastic multi-stage problem was not addressed due to the high complexity in solving (Golany et al. (2001), Helmrich et al., (2010)). Stochastic versions of LSPR face several challenges related to uncertainty and possible relationships between demand and returns. Van der Laan et al (1999) studied the correlation between demand and returns and its effect on the overall cost but also evaluate the case of an uncertain production lead time. The quantity of products to be remanufactured depends on the returns inventory level. Toktay et al (2000) propose a network flow formulation where returns are

considered as arrivals queue. Each node has specific parameters of arrival and length of service time.

Among the solution strategies for the stochastic version of LSPR, a reverse Wagner & Within algorithm is developed by Richter and Sombrutski (2000). Richter and weber (2001) extend the algorithm developed and consider cyclic manufacturing/remanufacturing continuous time periods with an optimal switching point. Similarly to Chung and al (2008) for static demand, DeCroix (2006) with a stochastic demand and returns generalizes the structure of the return process by considering a multi-echelon inventory system for the product recovery that will then be forwarded to the appropriate reprocessing option.

Solution approach is presented to decompose the original problem of each level into a series of problems with a single storage point and decisions concern the amount to be reprocessed and conveying to storage points.

In (Vercreane and al 2014), the authors studied the coordination of manufacturing, remanufacturing and returns acceptance control in a hybrid production-inventory system using a queuing control framework.

The uncertainty of returns, their variability and uncertainty associated with reprocessing time are other challenges that add to the complexity of the procurement process. Note that in practice, the returns are not necessarily correlated with the quantity sold. (DeCroix (2006)). The extension of the problem that addresses both options reprocessing and disposal is classified as NP-hard (Golany et al. (2001)). Kenne et al (2012) propose a control policy to minimize the sum of the holding and backlog costs for hybrid manufacturing–remanufacturing system under uncertainty. A stochastic dynamic programming based algorithm is developed to solve the optimal control problem within a dynamic continuous time context. We note in this study that uncertainty is related to failures events in the manufacturing/remanufacturing process and doesn't concern demand and returns.

The objective of this paper is to propose an extension of the stochastic versions of LSPRs found in literature and confirm the performance of our solution approach considering the preliminary results obtained in (EL FASSI and al 2014). Our main contribution consists in adapting the SDDP method to deal with a stochastic lot sizing problem where demand and returns evolve randomly. It was exclusively implemented for hydrothermal planning and other related power systems problems. We identify only one study that applies SDDP for inventory-transportation problem. (Fhoula et al. (2013)). However, it could be applied directly only for linear problems. In the solution approach section an optimization step to transform the MILP to LP with fixing the optimal binary variables is proposed. Then the algorithm developed is explained and more details about all optimization steps are given in the solution approach section.

Finally, in the preliminary results section, the application of the SDDP method for two instances is reported and a comparative study between the global solution (SDDP) and optimal solutions for each scenario realization is conducted for large instance ( 300scenarios). The obtained results demonstrate the accuracy and robustness of the method even with a higher variability of demand.

## 2 PROBLEM DEFINITION

We consider a lot sizing problem of manufactured and remanufactured items within a supply system with returns and hybrid manufacturing/remanufacturing line as described in Figure 1. The hybrid line refers to OEM strategy when original equipment manufacturer decides to involve remanufacturing operations into the manufacturing line. Returns are sent to a specific storage point. Then, after reprocessing, remanufactured items are supposed to be as good as new ones and then stored in a serviceable storage point. The latter hypothesis is common in remanufacturing industry as for single use camera (see Matsumoto 2009). Time periods are considered to be discrete and we assume a limited global capacity instead of the unrealistic unlimited capacity hypothesis largely considered in LSPRs literature.

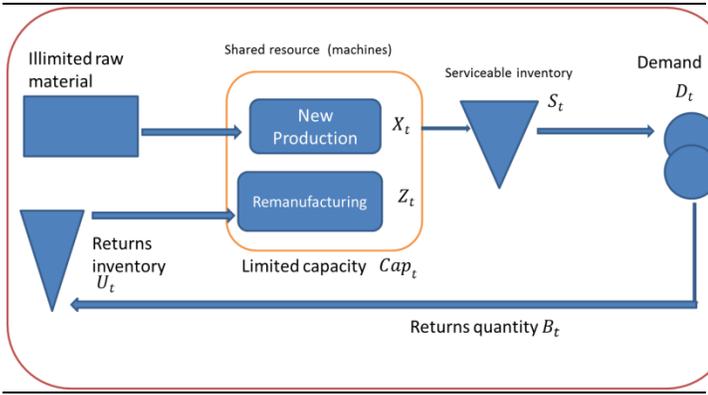


Fig. 1. The hybrid manufacturing/remanufacturing process

The lot sizing problem considered in this paper can be formally modeled using the following parameters and decision variables:

Parameters/data:

T: The planning horizon length (given in terms of number of discretized periods)

$D_t$  : Demand for new items in period  $t$

$B_t$  : Quantity of returned items in period  $t$

$P_t$  : Unit cost of manufacturing in period  $t$

$R_t$  : Unit cost of remanufacturing in period  $t$

$C_t$  : Fixed setup cost for a new production in period  $t$

$C'_t$  : Fixed setup cost for remanufacturing in period  $t$

$H_t$  : Unit cost for holding inventory of serviceable products in period  $t$

$H'_t$  : Unit cost for holding inventory of returned products in period  $t$

$M_t$  : Unit manufacturing capacity in period  $t$

$M'_t$  : Unit remanufacturing capacity in period  $t$

$L'_t$  : Unit setup remanufacturing capacity in period  $t$

$L_t$  : Unit setup manufacturing capacity in period  $t$

$G_t$  : Global line capacity in period  $t$

Decision variables:

$X_t$  : Quantity of manufactured items in period  $t$

$Z_t$  : Quantity of remanufactured items in period  $t$

$S_t$  : Serviceable inventory at the end of period  $t$

$U_t$  : Returns inventory at the end of period  $t$

$Y_t$  : Binary variable equal to 1 if a new production lot is launched; and to 0 otherwise

$Y'_t$  : Binary variable equal to 1 if a remanufacturing lot is launched; and to 0 otherwise

Mathematical formulation

$$\text{Min } \sum_{t=1}^T (P_t X_t + R_t Z_t + H_t S_t + H'_t U_t + C_t Y_t + C'_t Y'_t)$$

Subject to

$$S_t = S_{t-1} + X_t + Z_t - D_t; \forall t \in T$$

$$U_t = U_{t-1} + B_t - Z_t; \forall t \in T$$

$$M_t X_t + M'_t Z_t + L'_t Y'_t + L_t Y_t \leq G_t; \forall t \in T$$

(4)

$$X_t \leq G_t Y_t \quad ; \forall t \in T$$

$$Z_t \leq G_t Y'_t \quad ; \forall t \in T$$

$$S_t, U_t, X_t, Z_t \geq 0 \quad ; \forall t \in T$$

$$Y'_t, Y_t \in \{0,1\} \quad ; \forall t \in T$$

The objective function (1) minimizes the sum of manufacturing and remanufacturing costs, inventory costs, and setup costs over the periods. Equations (2) and (3) are the balance flows constraints for serviceable and returns inventory, respectively. Equation (4) is the global capacity constraint for each period including the capacity of manufacturing, remanufacturing and setups. Equations (5) and (6) link X and Y variables for manufactured and remanufactured products, respectively. Finally equations (7) and (8) are the non-negativity and binary constraints for variables definition.

The problem considered in this paper assumes that both demand for serviceable products  $(D_t)_{t=1...T}$  and returns  $(B_t)_{t=1...T}$  are stochastic. It is assumed that the random data process is stagewise independent. The main contribution of this paper is to propose a solution approach for solving this stochastic dynamic lot sizing problem with returns. The proposed approach is based on the Stochastic Dual Dynamic Programming (SDDP) initially developed by Pereira and Pinto (1991) as an extension to dynamic programming to solve multi-stage hydrothermal planning problems under water inflow uncertainty for large and complex systems. To the best of our knowledge, SDDP was exclusively implemented for hydrothermal planning and other related power systems problems. This is the first time that the SDDP method is applied for stochastic lot sizing problems. As will be explained in next section, the SDDP method is suitable for pure linear programming models since it uses dual multipliers. In our context, the problem is formulated as a Mixed Integer Programming (MIP) model which precludes using the SDDP method in its classic form. Another contribution of the paper is to propose an SDDP-based approach to solve dynamic stochastic problems that are modelled with MIPs rather than pure LPs.

### 3 SOLUTION APPROACH

The SDDP was first developed by Pereira and Pinto (1991) to solve multistage hydrothermal planning problems. Since then, the method was only implemented for solving hydrothermal planning and other related power systems problems presenting some data uncertainty. The basic idea of SDDP is to approximate the expected cost to go functions of stochastic dynamic programming by appropriate cuts obtained from the dual solutions of the optimization problem at each stage. We refer the reader to the paper by Pereira and Pinto (1991) for more details on the SDDP approach.

Using dual multipliers assume the optimization problem at each stage is modelled as a pure LP. However, the problem addressed in this paper needs both binary variables to model lot launching decisions and continuous variables to determine the quantity produced for each lot. In the following, we propose a two-step SDDP-based solution approach to circumvent such a difficulty.

First of all, let us assume that  $(Y_t)_{t=1..T}$  and  $(Y'_t)_{t=1..T}$  variables are fixed to values  $(\bar{Y}_t)_{t=1..T}$  and  $(\bar{Y}'_t)_{t=1..T}$ , respectively (by a first step procedure that we will explain in details later). Hence model (1)-(8) becomes a pure linear model and the classical SDDP method can be used to solve it. In the following, we give the main ingredients of the SDDP method adapted for the multistage stochastic lot sizing problem with returns addressed in this paper.

#### Overview of the SDDP method

If the stock levels of new and returned items are known at the beginning of each stage  $t$  (that is values of variables  $S_{t-1}, U_{t-1}$  are known), the lot sizing problem with returns can be represented as a stochastic dynamic programming recursion as follows:

$$\alpha_t(S_{t-1}, U_{t-1}) = E_{\tilde{D}_t, \tilde{B}_t} \left\{ \text{Min}_{X_t, Z_t} F_t(X_t, Z_t) + \alpha_{t+1}(S_t, U_t) \right\} \quad (9)$$

subject to

$$S_t = S_{t-1} + X_t + Z_t - \tilde{D}_t \quad (10)$$

$$U_t = U_{t-1} + \tilde{B}_t - Z_t \quad (11)$$

$$M_t X_t + M'_t Z_t \leq G_t - L'_t \bar{Y}'_t - L_t \bar{Y}_t \quad (12)$$

$$X_t \leq G_t \bar{Y}_t \quad (13)$$

$$Z_t \leq G_t \bar{Y}'_t \quad (14)$$

$$S_t, U_t, X_t, Z_t \geq 0 \quad (15)$$

for all  $t=T, T-1, \dots, 1$ ; for all  $S_{t-1}, U_{t-1}$ , where  $E(\cdot)$  denotes the expected value under stochastic demand and returns,  $\alpha_t(S_t, U_t)$

is the expected total cost from state  $(S_{t-1}, U_{t-1})$ ,  $F_t(X_t, Z_t)$  is the immediate cost associated with decisions  $X_t$  and  $Z_t$ .

#### State variables

The state variables describe the state of the general system at a given stage. In our case, the system is described by the inventory level of new items  $S_{t-1}$  and the inventory level of remanufactured items  $U_{t-1}$  at the beginning of a period  $t$ .

#### Decision variables

The decision vector includes the quantity of manufactured items in period  $t$ ,  $X_t$  and the quantity of remanufactured items in period  $t$ ,  $Z_t$ .

#### Transition equations

The transition equations permit to relate the decisions made in the current period with the previous ones. In our case, the transition equations correspond to the balance flows of serviceable storage point (equation (10)) and returns storage point (equation (11)).

#### Immediate cost

The immediate cost for a period  $t$  gives the total cost incurred for period  $t$  given the decisions  $X_t$  and  $Z_t$  made at this stage. In our case, the total immediate (or direct) cost is given by:

$$\text{Min } P_t X_t + R_t Z_t + H_t S_t + H'_t U_t, \text{ subject to constraints (10)-(15).}$$

The originality of the SDDP method compared to other stochastic dynamic programming methods is that the expected cost-to-go function  $\alpha_{t+1}(S_t, U_t)$  at a stage  $t$  is approximated by adding adequate cuts to the sub-problem (9)-(15) considered at stage  $t$ . Hence, there is no need to discretize the values of the state variables at each stage as required for classic dynamic programming methods circumventing thus the curse of dimensionality limits of DP algorithms. This is done recursively until stage 1 where  $\alpha_1(S_0, U_0)$  represents the total expected cost over the entire planning horizon. The uncertainty in problem data is modeled by generating  $S$  scenarios for demand and returns. The SDDP method considers two main steps: the so-called "backward recursion" and "forward simulation". In the backward recursion,  $n$  trial values  $\{\hat{S}_{t,i}; \hat{U}_{t,i}\}_{t=1..T, i=1..n}$  are considered for the state variables at each stage. A series of sub-problems (9)-(15) are then solved recursively starting with the last period  $T$  and going back to period 1. Sub-problem (9)-(15) at stage  $t$  is solved to optimality with the trial value  $\{\hat{S}_{t-1,i}; \hat{U}_{t-1,i}\}_{t=1..T, i=1..n}$  for each scenario  $s=1..S$  (this done by replacing  $D_t$  and  $B_t$  by the values of demand and returns in scenario  $s$ ,  $D_{ts}, B_{ts}$ ). The corresponding dual multipliers  $\pi_{t-1,i,s}$  are considered and an expected value for dual multipliers is computed as  $\bar{\pi}_{t-1,i} = \sum_{s=1}^S \pi_{t-1,i,s}$ . This expected dual multiplier  $\bar{\pi}_{t-1,i}$  is used to construct a cut to approximate the cost-to-go function at stage  $t-1$ . Hence, the number of trials considered determines the number of cuts added at each stage to approximate the cost-to-go function of the previous stage.

The forward pass enables constructing good trial decisions that are in “the neighborhood of interesting points” (Pereira and Pinto (1991)). The idea is to generate trial values gradually over stages by considering the optimal solution of the approximate sub-problem (9)-(15) at stage  $t$  (that is sub-problem (9)-(15) with the additional cuts added until the current iteration to approximate the cost-to-go function at stage  $t$ ). Since  $n$  trial decisions are needed at each stage, a sample of  $n$  scenarios (among the  $S$  considered) is randomly chosen and the corresponding sub-problems solved.

An iteration of the SDDP algorithm consists of a backward recursion step followed by a forward simulation step. We will discuss now the stopping criteria of the SDDP algorithm. Let us consider the simplest stopping criterion which consists in stopping the algorithm where a pre-specified number of iterations, say  $K_{max}$ , is reached. One should observe that at each iteration  $k$  of the SDDP algorithm, a lower bound  $\underline{z}_k$  on the minimum true cost can be obtained from the backward recursion step and an estimation of the upper bound  $\bar{z}_k$  can be obtained from the forward pass. Roughly speaking, a lower bound is given by the optimal objective value of the approximate sub-problem at stage 1 given all the cuts added to approximate the cost-to-go function at stage 1 until iteration  $k$ . Observe that at the first stage, we assume that demand and returns for the first period are known with certainty. The upper bound is estimated by considering the average value of the total approximate costs over the trials (the sample of  $n$  scenarios among the  $S$ ). One of the common stopping criteria for SDDP is to stop the algorithm when the lower bound is in the confidence interval of the upper

bound. We refer the reader to the paper of Pereira and Pinto (1991) for more details.

### Fixing binary variables

In this section, we present an approach for fixing  $Y$  and  $Y'$  variables in a consistent way with regard to SDDP.

As already mentioned, the basic idea in the SDDP method is to approximate cost-to-go functions by exploiting the LP nature of the sub-problems and using the duality theory to add good cuts. In our case, to circumvent this difficulty, we assume that lot launching variables are fixed enabling thus to have pure LP sub-problems (9)-(15).

Our preliminary results prove that our approach yields good results.

Assume that the SDDP method to be applied use  $S$  scenarios. As explained in the previous section, both the forward and backward passes solve the sub-problems (9)-(15) with either a subset of scenarios (one scenario for each trial to be considered) or all scenarios. We propose to solve model (1)-(8) to optimality for each scenario  $s=1, \dots, S$ . The values  $\bar{Y}'_{ts}, \bar{Y}_{ts}$  to be considered in sub-problem (9)-(15) at stage  $t$  and for scenario  $s$  correspond to the optimal values obtained for these binary variables for scenario  $s$  when solving model (1)-(8).

### 4 THE GLOBAL EXPERIMENTAL FRAMEWORK FOR STOCHASTIC RESOLUTION OF MSSLP

We describe in this section the global experimental framework to prove the performance of SDDP (fig.2).

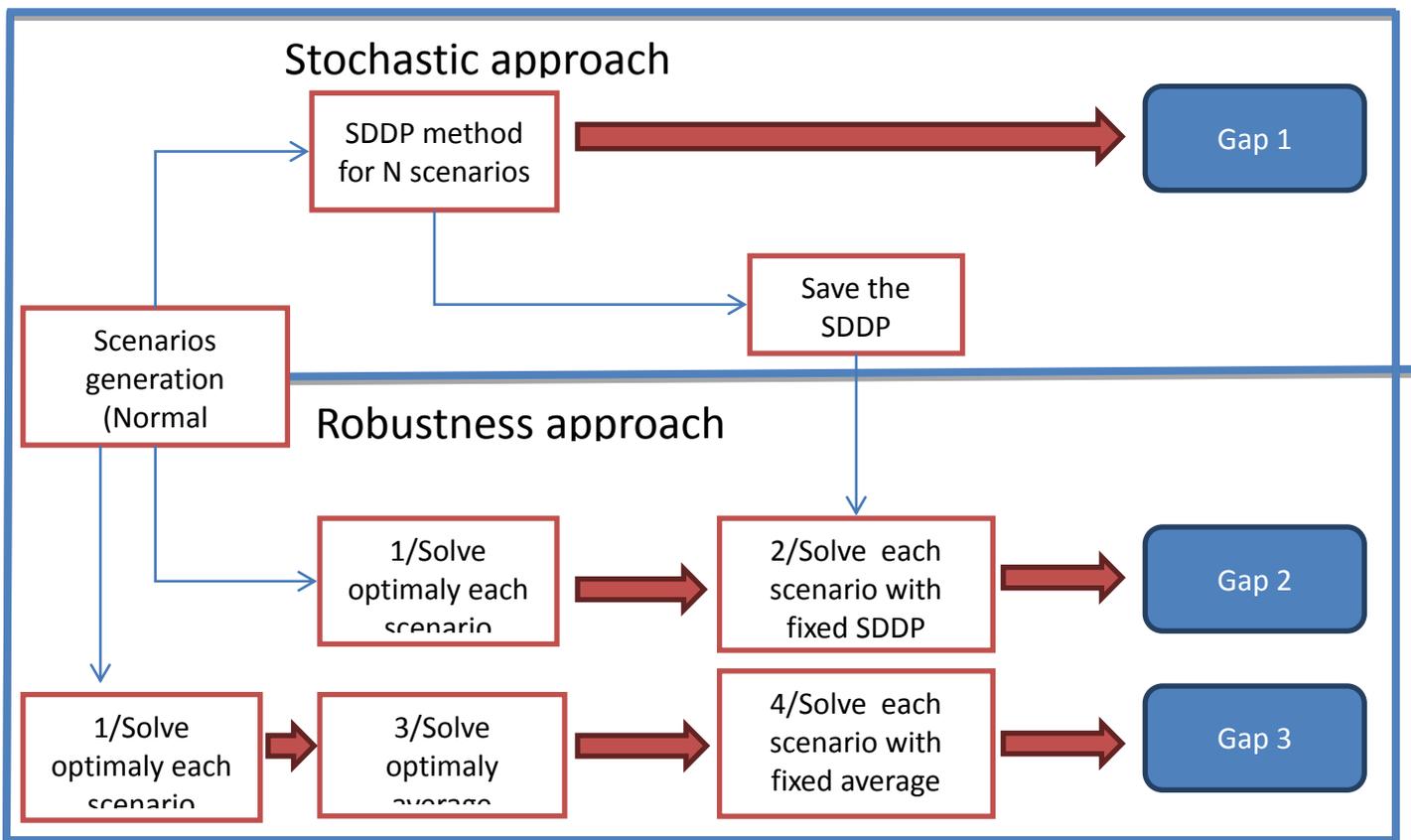


Fig 2. The global experimental framework for stochastic resolution of MSSLP

The fig 2 illustrates 4 ways to evaluate the performance and robustness of SDDP for the sampled scenarios. In the present work, we report only SDDP cost analysis and the gap 1.

After a scenario generation step, SDDP is then applied following the method steps as described in the solution approach section. The results of SDDP global cost for all scenarios are reported for each iteration.

When the convergence criterion is met, the best SDDP global cost (UUB) is obtained with the lower gap ratio which corresponds to the formula  $(UUB-LB)/LB$ . The upper upper bound (UUB) is the endpoint of the confidence interval of the upper bound. A high gap ratio refers to a high length of the confidence interval and consequently a low solution quality.

The fig.2 illustrates also the robustness approach considering deterministic data. After solving optimally each scenario, in theory, the SDDP global cost will be higher than the optimal objective of each one. The gap 1 measure the average deviation of SDDP cost considering all scenarios.

In the same logic, instead of comparing SDDP global cost and optimal objective value of each scenario, we try to test the SDDP solution vector. The latter corresponds to decision variables obtained by solving an average scenario when the convergence criterion is met (with the lower gap ratio). So we fix the X, Z solution vectors and solve model (1)-(8). Considering the fixed decision, and to ensure feasibility we add a backorder cost in the objective function to be able to meet demand for each scenario. Then we calculate the difference between the objective value with fixed SDDP decision and the optimal value for each scenario. The gap 2 is the average deviation.

In gap 3, we solve an average scenario and save the solution vector. We calculate the difference between the objective value with fixed decisions decision and the optimal value for each scenario. The objective is to prove that gap 2 is significantly lower than gap 3

Le comité d'organisation vous remercie, par avance, de votre compréhension et des efforts que vous fournirez pour cette rédaction.

## 5 PRELIMINARY RESULTS

In this section we report preliminary numerical experiments carried out for SDDP method and the MIP deterministic case by testing the impact of the demand and returns variability on the overall cost. A comparative study is also conducted to evaluate the performance of the proposed SDDP method.

As it is common in practice, we assume that the return costs and capacities are less than serviceable costs and new production capacities (Guide, 2000; Richter and Weber, 2001). In practice, the return rate is specific to the remanufacturing industry and depends on many factors like environmental legislation, the level of environmental consciousness and the return policy. To conduct our experiments we retained the approximate levels and rates of the electronic remanufacturing industry reported in Hauser & Lund (2008)). We consider 13 discrete time periods corresponding to tactical/operational planning horizon. Demand

and returns are normally distributed. Table 1 summarizes the data generation.

**Table 1: Data generation**

Forward parameters	Interval
Unit manufacturing Cost $P_t$	[2,3]
Fixed setup cost for manufacturing $C_t$	[20;22]
Unit cost for holding inventory of serviceable items $H_t$	[0,5 ;0,8]
Unit manufacturing capacity $M_t$	[2 ; 2 ]
Unit setup manufacturing capacity $L_t$	[2 ; 2 ]
Reverse parameters	Interval
Unit remanufacturing Cost , $R_t$	[1;1,5]
Fixed setup cost for remanufacturing $C'_t$	[9;12]
Unit cost for holding inventory of returned items $H'_t$	[0,2 ;0,3]
Unit remanufacturing capacity $M'_t$	[0,7 ;1]
Unit setup remanufacturing capacity $L'_t$	[0,7 ;1]

The global line capacity in period  $t$  is assumed to be larger than  $(D^* M_t + L_t)$  which represents the minimum quantity required to respond to demand.

All numerical tests are conducted on an Intel (R) Core i3. We use the Cplex 12.2 solver Concert technology/Visual C++ (2010).

We consider 2 instances (see Table 2) with 2 different demand profile. The perturbed demand corresponds to a higher variability from one time period to another. For each instance 300 scenarios are generated, the number of maximum iterations is fixed to 5 together with the aforementioned stopping criteria explained in section 3.

**Table 2: Instance generation**

Instances	Normal distribution parameters of demand and returns	Demand profile
1	Demand $N(100,30)$ and Returns $\sim N(40,5)$ .	Stationary demand
2	Demand $N(100,30)$ and Returns $\sim N(40,5)$ .	Perturbed demand

The preliminary results are intended to illustrate the performance of the proposed SDDP approach within a sensitivity analysis on demand mean and variability. The table 3 shows results for SDDP application for instance 1.

**Table 3: SDDP cost analysis**

Iteration	SDDP Lower Bound	SDDP value (UUB)	Gap	Gap ratio : GAP/LB	Standard deviation	CPU time (s)
1	0	0	0	0	0	
2	2237,3	2782,2	54	24,31	300.95	13,821
3	0	0	0	0	0	7
4	0	0	0	0	0	
5	0	0	0	0	0	

The obtained results reported in table 3 show that convergence criterion is met only in iteration 2, with a global SDDP cost of 2782, 22\$ and a gap of 544 and a gap ratio of 24, 31%. The latter is relatively high. This is can be explained by the high number of scenarios (300) considered and their variability. Although the gap ratio can be improved by increasing the number of iterations since the computational time is still low (13,82s per iteration). We executed the method for the same instance with 40 iterations and we obtained in iteration 8 an SDDP global cost of 2492, 96 and a gap ratio of 11, 2%. This can be explained by a better approximation of uncertainty since “good cuts” are added incrementally for each iteration. However a low improvement of the gap ratio is observed after iteration 8.

The previous observations are supported by additional experiments where the proposed SDDP method solutions are compared to the optimal ones. The scenarios problems are solved separately and the optimum value for each one is saved. Then, we compared the global solution obtained by SDDP method with the optimal objective value of each scenario. A relative average gap for each scenario is then calculated to evaluate the robustness and accuracy of SDDP method. Then we calculate the average optimality gap (gap1) of all relative averages for each instance and the corresponding standard deviation (table 4). The obtained results reported in table 4 show that when the variability of demand increases then the global cost of SDDP increases. However, the cost increase is not important considering the high variability of demand. Also we can observe that gap 1 for instance 2 is lower than instance1 even when variability is higher. This result demonstrates that even if the decision maker faces a lack of information about futures demand (or returns), he has a powerful decision tool that helps him to define robust strategies.

We can see that the mean of average optimality gaps is 12,44% which demonstrates that SDDP method permits to generates manufacturing/remanufacturing policies that are robust even for more than 300 scenarios.

**Table 4: SDDP method versus optimal decision for each scenario s**

Instance	SDDP value	Gap1	Standard deviation
1	2492, 96	12,63%	10,49%
2	<b>2622,27</b>	12,26%	10,33%
		<b>12,44%</b>	<b>10,41%</b>

In the present work, we presented a multi-period stochastic linear problem to formulate a hybrid manufacturing/remanufacturing system. We propose an SDDP method as a solution approach for the considered problem with extended results and larger instance for more 300 scenarios and 10 iterations. We also propose a global experimental framework to evaluate the accuracy of SDDP method. The results show that a higher variability in demand or returns affects the global cost but paradoxically the robustness is better for higher variability (gap1 for instance2). We obtain by SDDP an average optimality gap of 12,44% compared with optimal solutions for all scenarios. This gap value demonstrates within the instances considered, the performance of the method to give a good estimation for the decision maker to define robust manufacturing/remanufacturing strategies.

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