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Robust harvesting planning under random supply and demand

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***Abstract-* Harvesting planning is one of the most important tactical decisions in lumber supply chains. Harvesting areas in the forests are divided into different blocks with different types and quantities of raw materials (logs). The availability of raw materials in each block cannot be forecasted with certainty. On the other side, predicting the certain amount of the demand is impossible in this industry. As a consequence, it is necessary to consider uncertainty in the harvesting planning. In this paper, we propose a robust harvesting planning model under log supply and demand uncertainty. The proposed robust optimization model which has been formulated based on "price of robustness" provides some insights into the adjustment of the level of robustness of the harvesting plan over the planning horizon and protection against uncertainty.**

***Keywords-* Robust harvesting planning, Lumber supply chain, Uncertain supply and demand, Price of robustness**

1 Introduction

Despite the applicability of stochastic programming models in many planning problems, it has some limitation in solving real-world problems. Such models require full knowledge of the distributions of the uncertain data and such information is rarely available in practice, and a strategy based on erroneous inputs might be infeasible or exhibit poor performance when implemented. Moreover, these models are very large and difficult to solve for real instances. Such challenges in stochastic models have made robust optimization to receive attention. Robust optimization is one of the predominant approaches to solving linear optimization problems with uncertain data where there is not enough information about their probability distributions. The classical approach to robust optimization is to search for an optimal solution which has the property that the solution will satisfy all possible out comes. The robust optimization approach is categorized into static and dynamic models. When the decision maker must choose the strategy before the exact values of uncertain parameters are revealed, the robust solution is called static. In the other words, all decision variables are "here and now". In this case, the model does not allow for recourse action, and remedial action is occurred when the values of the random variables become known. The objective is typically to minimize the worst case cost, and the uncertainty can take two forms: (i) estimation errors for unknown parameters, and (ii) stochasticity of random variables (Bertsimas and Thiele (2006a)).

The first step in the robust optimization approach was taken by Soyster (1973). In this study, he proposed a linear optimization model to construct a solution that is feasible for all data that belong to a convex set. This approach was further developed by Ben-Tal and Nemirovski (1998, 1999); Ben-Tal et al. (2004), El Ghaoui and Lebret (1997), El Ghaoui et al. (1998), Bertsimas and Sim (2004), Bertsimas and Thiele (2006b).

Ben-Tal et al. (2010) present the main strategy of the robust optimization approach. In this approach, first the original uncertain problem is reformulated to its robust counterpart. Then the robust counterpart model may be provided as an explicit and short system of convex inequalities. The latter is also called a “computationally tractable” representation of the robust counterpart. By such representing of the robust counterparts of each constraint of the uncertain problem, it will be possible to reformulate the robust counterpart of the original uncertain problem as a linear minimization one under a finite and short system of explicit convex constraints which are computationally tractable.

Bertsimas and Sim (2004) propose a solution approach for a linear mathematical model with an uncertain coefficient matrix. By assuming interval uncertainty, their approach provides a robust solution whose level of conservatism can be flexibly adjusted in terms of probabilistic bounds for constraint violation. They explore the current status of static robust optimization for linear programming problems. This approach finds a solution over all time periods. They define a predetermined budget of uncertainty for every constraint in order to provide an optimal solution that guarantee feasibility for all admissible data realization of uncertain parameters at a given probability (confidence level).

When the planning is dynamic, it is reasonable to expect that better solutions can be found as we can dynamically adjust the planning when more information is known. This is called a dynamic robust solution. In the dynamic planning situation it is assumed that there are two sets of variables. One set must be determined before all the parameters are determined, and the other set of variables model future decisions that need not be determined until a later stage. Ben-Tal et al. (2004) introduce a computationally tractable robust formulation for the special case when the future decision variables can be expressed with an affine function of the uncertainty set. Although they observe many situations where this assumption holds, it is not an easy task to verify if the problem at hand satisfies all the requirements. The method has no flexibility in elaborations with uncertainty sets, since a minor adjustment could change the robust counterpart into an intractable formulation. Bertsimas and Thiele (2006b) present the robust optimization method for inventory management problem under demand uncertainty over the planning horizon. By assuming an interval uncertainty for demand, they develop the robust counterpart for inventory control problem in dynamic settings. In the inventory problems, the constraints depend on the time period, consequently the uncertainty set will depend on the time period as well. The uncertainty is modeled as the cumulative demand up to time t. This motivates defining a sequence of budgets of uncertainty Γ_t, $t = 0, ..., T-1$, rather than using a single predetermined budget explained in the static case. As only one new source of uncertainty is revealed at any time, the budgets do not increase by more than one at each time period.

Bertsimas and Caramanis (2010) approach a more general problem where the uncertainty set may be a general polytope. In the solution approach, they use a partitioning of the uncertainty set and find a static robust solution for each partition. In a later stage, without uncertainty, at least one of the static solutions fulfills the now realized parameters, and the best static solution is selected for implementation. The difficulty with this approach is to select a well performing partitioning so that the static robust solutions are reasonable, while at the same time keeping the number of partitions low for the sake of efficiency.

Adida and Perakis (2006) introduce a robust optimization model to dynamic pricing and inventory control. They propose a linear function such as $gt + b$ for a time-dependent budget of uncertainty where $g, b \geq 0$ and $g \leq 1$ guarantee to avoid very conservative values for the budget of uncertainty and control the level of conservatism. There are more papers in the literature worked on the dynamic robust optimization where the budget of uncertainty for each period is generated randomly such as Bienstock and ÖZbay (2008) and Li and Ierapetritou (2008). Generally, these approaches generate budgets of uncertainty in the scale of $\sqrt{t+1}$. Alvarez and Vera (2014) present the application of robust optimization methodology to a sawmill planning problem. They consider an equal amount of budget of uncertainty to represent the grade of robustness to each constraint.

Harvesting planning is one of the most important decisions in the lumber supply chain. Two main operations in the forests are harvesting and forwarding. The main important tactical decisions in the forests are the harvesting area (block) selection and bucking over the planning horizon (Bredström et al. (2010)). Wood procurement models can be tracked back to the early 1960s. Since that time, several models have been developed to address different aspects of wood procurement (Beaudoin et al. (2006)). Some of these models have been designed for specific activities such as skidding or transportation (Carlsson et al. (1999); Wightman and Jordan (1990)). Beaudoin et al. (2006) proposed a deterministic model for forest tactical planning. They also assessed the impact of uncertainty into their model and evaluated these uncertainties under alternative tactical scenarios by the aid of simulation. Other models tried to integrate several forest planning decisions in a single model, in order to capture possible synergies between them. As an instance, Burger and Jamnick (1995) integrated harvesting, storage, and transportation decisions. Andalaft et al. (2003) integrated harvesting and road-building decisions. Karlsson et al. (2004) presented an optimization model for annual harvest planning. Their model includes transportation planning, road maintenance decisions, and control of storage in the forest and at terminals in mills. Bredström et al. (2010) formulated a mixed-integer programming (MIP) model to integrate the assignment of machines and harvest teams to harvesting blocks. They proposed a two stage methodology such that the first one solves the assignment and the second one tries to schedule. Dems et al. (2014) developed a MIP model for annual timber procurement planning with considering bucking decisions in order to minimize the operational costs such as harvesting, transportation, and inventory costs. In their proposed procurement planning model, they considered a multi-period, multi-product, multiple blocks and multi-mill setting. Chauhan et al. (2011) proposed an integrated approach for harvesting, bucking, and transportation decisions. They assumed a multi-product and multi-mill setting in a single period planning horizon. To minimize the harvesting and transportation costs in the forest, they developed a heuristic algorithm based on the column generation approach. Sanei Bajgiran et al. (2014) proposed a mixed-integer-programming model to address harvesting, procurement, production, distribution, and sale decisions in lumber supply chain in integrated and two decoupled scheme in deterministic environment. They evaluated the benefit of the integrated model by comparing it with decoupled models in terms of total revenue and costs based on a realistic-scale industrial case study. The proposed integrated model in their work was too complex, thus they proposed a Lagrangian Relaxation (LR) Heuristic algorithm to overcome this complexity.

To summarize, the paper contribution is twofold. The first contribution of this paper is to explore robust harvesting model in lumber supply chains under supply and demand uncertainties. We present a comprehensive robust model to deal with uncertain log supply and demand which affect the right hand side, constraints, and the objective function coefficients simultaneously. Furthermore, the proposed robust optimization model which has been formulated based on "price of robustness" provides a way to adjust the level of robustness of the harvesting plan over the planning horizon and protection against uncertainty. To the best of our knowledge, the robust optimization approach applied in this paper, has not been applied to the harvesting planning model in the literature.

The paper remainder is organized as follows. The robust optimization approach is presented in Section 2. The robust harvesting planning model is provided in Section 3. Finally, the conclusions and future works are presented in Sections 4.

2 The robust optimization approach

In this paper, we rely on the robust optimization approach developed by Bertsimas and Sim (2004) for linear programming problems. Let's consider the following Linear Programming (LP) model:

$$
\begin{aligned}
& Max\ Z = cx \\
& \tilde{a}x \leq b \\
& l \leq x \leq u
\end{aligned} \tag{1}
$$

where some parameters of the coefficient matrix (a_{ij}) are uncertain. In addition, each uncertain parameter $(\tilde{a}_{ij})$ takes a value according to a symmetric distribution with mean equals to the nominal value $(\bar{a}_{ij})$ in the interval $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$. Furthermore, they define a parameter Γ, budget of uncertainty, for every constraint. This parameter is not necessarily integer and takes a value in the interval $[0, |J_i|]$, where J_i is the set of uncertain parameters in the ith constraint. Finally, they propose a linear robust counterpart to protect against all cases that $\lfloor \Gamma_i \rfloor$ coefficients of set J_i are permitted to change, and one coefficient (a_{it_i}) changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{ij}$. In order to guarantee feasibility, they consider a protection function for every constraint i which are named $\beta(x, \Gamma_i)$ and are equal to:

$$
\beta(x, \Gamma_i) = \max_{\{ S_i \cup t_i | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i \}} \left\{ \sum_{j \in J_i} \hat{a}_{ij} |x_j| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} |x_{t_i}| \right\} \tag{2}
$$

Therefore, model (1) can be rewritten as model (3):

$$
\begin{aligned}
& Max\ Z = cx \\
& s.t.: \sum_j a_{ij}x_j + \max_{\{S_i \cup t_i | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \\
& \left\{ \sum_{j \in J_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} y_{t_i} \right\} \le b_i \quad \forall i \\
& -y_j \le x_j \le y_j \quad \forall j \\
& l_j \le x_j \le u_j \quad \forall j \\
& y_j \ge 0 \quad \forall j
\end{aligned} \tag{3}
$$

Finally, they prove that model (3) has a robust counterpart as follows which is a general robust counterpart where the constraint coefficients are uncertain in static robust optimization problems.

$$
\begin{aligned}
& Max\ Z = cx \\
& s.t.: \sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{j \in J_i} p_{ij} \le b_i \quad \forall i \\
& z_i + p_{ij} \ge \hat{a}_{ij} y_j \quad \forall i, j \in J_i \\
& -y_j \le x_j \le y_j \quad \forall j \\
& l_j \le x_j \le u_j \quad \forall j \\
& y_j \ge 0 \quad \forall j \\
& z_i \ge 0 \quad \forall i \\
& p_{ij} \ge 0 \quad \forall i, j \in J_i
\end{aligned} \tag{4}
$$

For the robust counterpart (4), this approach provides an effective method to determine probability bounds for the constraint violation. The probability bound that the ith constraint is violated can be approximated as follows:

$$
pr(\sum_j \tilde{a}_{ij} x_j^* > b_i) \le 1 - \phi(\frac{\Gamma_i - 1}{\sqrt{|J_i|}}) \tag{5}
$$

where $\phi(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} \exp(-\frac{y^2}{2}) dy$ is the cumulative standard normal distribution function for all i, and x_j^* be the optimal solution of the robust optimization problem.

3 Robust harvesting planning

In the harvesting planning model, there are two uncertain parameters; uncertain log demand and supply. Thus, in this part we propose a robust counterpart for the harvesting planning model encountering with uncertain right-hand side, uncertain constraint and objective function coefficient. Notice that in the robust model, we aim at considering intervals for uncertain parameters (demand and supply). Thus, the robust method will provide an estimate of the worst case scenario over all periods in addition to the adjustment of the level of robustness of the harvesting plan over the planning horizon and protection against uncertainty.

The mathematical model of harvesting planning with considering uncertain demand and supply ($\tilde{d}_{rm,t}^{bl}$ and $\tilde{v}_{rm,bl}$) is proposed in this section. This model tries to minimize the harvesting, inventory, and stumpage costs. The harvesting decisions involve the blocks where the harvesting should occur ($H_{bl,t}$) as well as the proportion of the harvested blocks in different periods of the planning horizon ($y_{bl,t}$). The inventory of each raw material in each block in different periods ($I_{rm,bl,t}$) is the other decisions in the harvesting model. Constraint (7) formulates the inventory balance of raw materials in each block. Constraint (8) represents the final inventory of raw materials in each block. Constraint (9) ensures that the harvested proportion of a block do not exceed the availability of logs in that block. Constraint (10) describes that if harvesting occurs on a block then we can ensure that raw materials from that block are available. Constraints (11) and (12) correspond to the maximum number of harvesting and maximum number of blocks in which harvesting can occur, respectively. Constraints (13) corresponds to harvesting capacity in the blocks at each period.

$$
\begin{aligned}
Min\ Z = & \sum_{bl \in BL} \sum_{t \in T} c_{blt}^H y_{bl,t} (\sum_{rm \in RM} \tilde{v}_{rm,bl}) \\
& + \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in T} \tilde{v}_{rm,bl} f_{rm,bl,t} y_{bl,t} \\
& + \sum_{rm \in RM} \sum_{bl \in BL} \sum_{t \in T} c_{rm,bl,t}^S I_{rm,bl,t}
\end{aligned} \tag{6}
$$

Subject to:

$$
I_{rm,bl,t} = I_{rm,bl,t-1} - \tilde{d}_{rm,t}^{bl} + \tilde{v}_{rm,bl} y_{bl,t} \quad \forall rm, bl, t \tag{7}
$$

$$
I_{rm,bl,T} = 0 \quad \forall rm, bl \tag{8}
$$

$$
\sum_{t \in T} y_{bl,t} \le 1 \quad \forall bl \tag{9}
$$

$$
y_{bl,t} \le H_{bl,t} \quad \forall bl, t \tag{10}
$$

$$
\sum_{t \in T} H_{bl,t} \le l_{bl} \quad \forall bl \tag{11}
$$

$$
\sum_{bl \in BL} H_{bl,t} \le n_t \quad \forall t \tag{12}
$$

$$\sum_{bl \in BL} (y_{bl,t} \sum_{rm \in RM} \tilde{v}_{rm,bl}) \leq b_t^H \quad \forall t \tag{13}$$

$$y_{bl,t}, I_{rm,bl,t} \geq 0, H_{bl,t} \in \{0,1\} \, \forall bl, rm, t \tag{14}$$

As explained earlier, in the harvesting planning model, there are two types of uncertainty including, $\tilde{d}^{bl}_{rm,t}$ and $\tilde{v}_{rm,bl}$. $\tilde{d}^{bl}_{rm,t}$ is an uncertain log demand with the nominal value of $\bar{d}^{bl}_{rm,t}$, and $\tilde{v}_{rm,bl}$ is an uncertain log supply with a nominal value of $\bar{v}_{rm,bl}$. The uncertain log demand is assumed symmetric and time-dependent. This random variable ($\tilde{d}^{bl}_{rm,t}$) takes values in the interval $[\bar{d}^{bl}_{rm,t} - \hat{d}^{bl}_{rm,t}, \bar{d}^{bl}_{rm,t} + \hat{d}^{bl}_{rm,t}]$. Then, the scale deviation $z^{bl}_{rm,t}$ belonging to $[-1,1]$ is considered for $\tilde{d}^{bl}_{rm,t}$ from its nominal value as $z^{bl}_{rm,t} = (\tilde{d}^{bl}_{rm,t} - \bar{d}^{bl}_{rm,t})/\hat{d}^{bl}_{rm,t}$. Thus, we can also write $\tilde{d}^{bl}_{rm,t} = \bar{d}^{bl}_{rm,t} + \hat{d}^{bl}_{rm,t} z^{bl}_{rm,t}$.

The other uncertain parameter is $\tilde{v}_{rm,bl}$ assumed time independent and taken values in the interval $[\bar{v}_{rm,bl} - \hat{v}_{rm,bl}, \bar{v}_{rm,bl} + \hat{v}_{rm,bl}]$. We consider the scale deviation of $w_{rm,bl}$ from its nominal value as $w_{rm,bl} = (\tilde{v}_{rm,bl} - \bar{v}_{rm,bl})/\hat{v}_{rm,bl}$ that belongs to $[-1,1]$. Again, the random supply might be rewritten as $\tilde{v}_{rm,bl} = \bar{v}_{rm,bl} + w_{rm,bl}\hat{v}_{rm,bl}$.

The first constraint in the abovementioned harvesting planning model formulates the inventory balance of raw materials in each block including both uncertain parameters (supply and demand). The main decision variable in this constraint is the proportion of harvested block ($y_{bl,t}$). The quantity of the inventory in each block is the state variable ($I_{rm,bl,t}$). In the static robust optimization approach, all decision variables are "her and now" and there is not any possibility for recourse actions. Inspired by Bertsimas and Thiele (2006b), it is possible to remove the state variables and cumulate the effects of uncertain parameters by rewriting constraint (7) in the following closed loop equation which is the evolution of the inventory over time.

$$I_{rm,bl,t} = I_{rm,bl,0} + \sum_{s=1}^{t} (\tilde{v}_{rm,bl} y_{bl,s} - \tilde{d}^{bl}_{rm,s}) \quad \forall rm, bl, t \tag{15}$$

As the inventory quantity (state variable) also exists in the objective function, and we already tried to remove them from our constraints, it is possible to consider the total amount of the storage cost as a constraint such as (16), and substitute term $c^S_{rm,bl,t} I_{rm,bl,t}$ in the objective function by $HH_{rm,bl,t}$ which is defined as the storage cost at the end of period t.

$$c^S_{rm,bl,t}(I_{rm,bl,0} + \sum_{s=1}^{t} (\tilde{v}_{rm,bl} y_{bl,s} - \tilde{d}^{bl}_{rm,s})) \leq HH_{rm,bl,t} \quad \forall rm, bl, t \tag{16}$$

Next, we are looking for the robust counterpart of constraint (16) such that the inventory cost over all realizations of uncertain demand and log supply is minimized. In other words, we are aim for minimizing the maximum amount of the right-hand side of constraint (16) over the set of all admissible realization of uncertain log demand and supply. To do this, we should find a feasible solution with considering the worst cases for uncertain parameters. The worst case for uncertain supply ($\tilde{v}_{rm,bl}$) which is a time-independent parameter is $\bar{v}_{rm,bl} + \hat{v}_{rm,bl}$, and the worst case for uncertain demand ($\tilde{d}^{bl}_{rm,t}$), the time-dependent parameter, is calculated based on the following protection function. In reality, it is unlikely that all uncertain demand changes, thus we assume a predetermined amount of log demand in each block is unknown, and define $\Gamma^{bl}_{rm,t}$ as the total number of uncertain demand for a given rm, bl until period t ($\Gamma^{bl}_{rm,t} \in [0,t]$).

$$\begin{aligned}
&Maximize \sum_{s=1}^{t} \hat{d}^{bl}_{rm,s} z^{bl}_{rm,s} \\
&s.t. \quad \sum_{s=1}^{t} z^{bl}_{rm,s} \leq \Gamma^{bl}_{rm,t} \\
&0 \leq z^{bl}_{rm,s} \leq 1 \quad \forall s \leq t
\end{aligned} \tag{17}$$

This protection function is equivalent to the following optimization problem as its dual problem.

$$\begin{aligned}
&Minimize \quad \lambda^{bl}_{rm,t} \Gamma^{bl}_{rm,t} + \sum_{s=1}^{t} \theta_{rm,bl,t,s} \\
&s.t. \quad \lambda^{bl}_{rm,t} + \theta_{rm,bl,t,s} \geq \hat{d}^{bl}_{rm,s} \quad \forall rm, bl, t, \forall s \leq t \\
&\lambda^{bl}_{rm,t}, \theta_{rm,bl,t,s} \geq 0 \quad \forall rm, bl, t, \forall s \leq t
\end{aligned} \tag{18}$$

where $\lambda^{bl}_{rm,t}$ and $\theta_{rm,bl,t,s}$ are the dual variables corresponding to the constraints of protection function (17). By substitution of the dual objective function and its constraint instead of protection function (17), the following constraints are concluded. Moreover, because the inventory amount at the end of each period is always greater or equal to zero, constraint (21) should be added to the robust model which guarantees the positive amount of inventory for all possible data realization of random supply and demand.

$$HH_{rm,bl,t} \geq c^S_{rm,bl,t}(I_{rm,bl,0}+ \sum_{s=1}^{t}((\bar{v}_{rm,bl}+\hat{v}_{rm,bl})y_{bl,s}-\bar{d}^{bl}_{rm,s})+ \lambda^{bl}_{rm,t}\Gamma^{bl}_{rm,t}+\sum_{s=1}^{t}\theta_{rm,bl,t,s}) \quad \forall rm,bl,t \tag{19}$$

$$\lambda^{bl}_{rm,t}+\theta_{rm,bl,t,s} \geq \hat{d}^{bl}_{rm,s} \quad \forall rm,bl,t,\forall s \leq t \tag{20}$$

$$(I_{rm,bl,0}+\sum_{s=1}^{t}((\bar{v}_{rm,bl}-\hat{v}_{rm,bl})y_{bl,s}-\bar{d}^{bl}_{rm,s})- \lambda^{bl}_{rm,t}\Gamma^{bl}_{rm,t}-\sum_{s=1}^{t}\theta_{rm,bl,t,s}) \geq 0 \quad \forall rm,bl,t \tag{21}$$

$$\lambda^{bl}_{rm,t},\theta_{rm,bl,t,s} \geq 0 \quad \forall rm,bl,t,\forall s \leq t \tag{22}$$

As it can be observed in model (6)-(14), the first two terms in the objective function contain uncertain log. In order to obtain the robust counterpart, first, we consider the uncertain part of objective function as a constraint, then we develop the protection function for each constraint, and finally it is possible to obtain the robust counterpart of harvesting planning model (R-HP). Hence, we substitute $(\sum_{bl\in BL}\sum_{t\in T}c^H_{blt}y_{bl,t}(\sum_{rm\in RM}\tilde{v}_{rm,bl})$ by π_1, and $\sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T}\tilde{v}_{rm,bl}f_{rm,bl,t}y_{bl,t})$ by π_2 in the objective function and consider the following constraints in the harvesting planning model.

$$\sum_{bl\in BL}\sum_{t\in T}c^H_{blt}y_{bl,t}(\sum_{rm\in RM}\tilde{v}_{rm,bl}) \leq \pi_1 \tag{23}$$

$$\sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T}\tilde{v}_{rm,bl}f_{rm,bl,t}y_{bl,t} \leq \pi_2 \tag{24}$$

Again, the robust optimization approach tries to find a feasible solution in constraints (23) and (24) over the set of all possible uncertain supplies. Moreover, it is unlikely that all uncertain supply changes simultaneously, thus we consider the budgets of uncertainty $\Gamma^{\pi_1}_{bl}$ and Γ^{π_2} in order to adjust the total amount of uncertain log supply in each block. Intuitively from the previous part, we should consider the worst case for $\tilde{v}_{rm,bl}$ based on the mentioned budgets of uncertainty. As a consequence, we propose the following protection functions for constraints (23) and (24) in order to calculate the worst cases. Notice that $y^*_{bl,t}$ is considered as the optimal solution of model (6)-(14).

$$\begin{aligned} &Maximize \sum_{bl\in BL}\sum_{t\in T}c^H_{blt}y^*_{bl,t}(\sum_{rm\in RM}\hat{v}_{rm,bl}w_{rm,bl}) \\ &s.t. \quad \sum_{rm\in RM}w_{rm,bl} \leq \Gamma^{\pi_1}_{bl} \quad \forall bl \\ &0 \leq w_{rm,bl} \leq 1 \quad \forall rm,bl \end{aligned} \tag{25}$$

$$\begin{aligned} &Maximize \sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T}\hat{v}_{rm,bl}f_{rm,bl,t}y^*_{bl,t}w_{rm,bl} \\ &s.t. \quad \sum_{rm\in RM}\sum_{bl\in BL}w_{rm,bl} \leq \Gamma^{\pi_2} = \sum_{bl\in BL}\Gamma^{\pi_1}_{bl} \\ &0 \leq w_{rm,bl} \leq 1 \quad \forall rm,bl \end{aligned} \tag{26}$$

The robust counterpart of these two constraints is equivalent to find a harvesting plan such that the harvesting and stumpage costs over all realizations of log supply is minimized. Afterwards, by considering $z^{\pi_1}_{bl}$, z^{π_2}, $\theta^{\pi}_{rm,bl}$, and $\beta^{\pi}_{rm,bl}$ as the dual variables of the related protection functions (25) and (26), and $\Gamma^{\pi_1}_{bl}$ and Γ^{π_2} as the budget of uncertainties corresponding to constraints (23) and (24), the following constraints should be added to the R-HP.

$$\sum_{bl\in BL}\sum_{t\in T}c^H_{blt}y_{bl,t}(\sum_{rm\in RM}\bar{v}_{rm,bl})+ \sum_{bl\in BL}\Gamma^{\pi_1}_{bl}z^{\pi_1}_{bl}+\sum_{rm\in RM}\sum_{bl\in BL}\theta^{\pi}_{rm,bl} \leq \pi_1 \tag{27}$$

$$\sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T}\bar{v}_{rm,bl}f_{rm,bl,t}y_{bl,t}+ \Gamma^{\pi_2}z^{\pi_2}+\sum_{rm\in RM}\sum_{bl\in BL}\beta^{\pi}_{rm,bl} \leq \pi_2 \tag{28}$$

$$z^{\pi_1}_{bl}+\theta^{\pi}_{rm,bl} \geq \sum_{t\in T}c^H_{blt}y_{bl,t}\hat{v}_{rm,bl} \quad \forall rm,bl \tag{29}$$

$$z^{\pi_2}+\beta^{\pi}_{rm,bl} \geq \sum_{t\in T}\hat{v}_{rm,bl}f_{rm,bl,t}y_{bl,t} \quad \forall rm,bl,t \tag{30}$$

$$z^{\pi_1}_{bl},z^{\pi_2},\theta^{\pi}_{rm,bl},\beta^{\pi}_{rm,bl} \geq 0 \quad \forall rm,bl \tag{31}$$

Constraint (13) in model (6)-(14) is another constraint faced with uncertain supply ($\tilde{v}_{rm,bl}$). This constraint indicates that the total quantity that might be harvested must not exceed the harvesting capacity at each period for all realization of $\tilde{v}_{rm,bl}$. Thus, the total quantity of harvesting should equal to the harvesting capacity at the worst case. In order to find the worst case scenario for all $\tilde{v}_{rm,bl}$, the harvesting quantity should be maximized. Hence, the following protection function might be developed for a given t by the budget of uncertainty Γ^v_{bl}

which indicates the maximum amount of uncertain supply in each block. Again, $y^*_{bl,t}$ is the optimal solution of model (6)-(14).

$$
\begin{aligned}
&Maximize \sum_{bl\in BL}(y^*_{bl,t}\sum_{rm\in RM}\hat{v}_{rm,bl}w_{rm,bl})\\
&s.t. \quad \sum_{rm\in RM} w_{rm,bl} \le \Gamma^v_{bl} \quad \forall bl\\
&0 \le w_{rm,bl} \le 1 \quad \forall rm, bl \qquad (32)
\end{aligned}
$$

For the above model, we define z^v_{bl} and $\theta^v_{rm,bl}$ as the dual variables corresponding to constraints of model (32). By applying the same approach explained earlier, we can substitute the dual objective function and its constraints in the R-HP model in order to find a robust solution which is feasible for all realization of uncertain supply ($\tilde{v}_{rm,bl}$) according to a predetermined budget of uncertainty (Γ^v_{bl}). As a consequence, the following constraints are concluded.

$$
\begin{aligned}
&\sum_{bl\in BL}(y_{bl,t}\sum_{rm\in RM}\bar{v}_{rm,bl}) + \sum_{bl\in BL}\Gamma^v_{bl}z^v_{bl} +\\
&\sum_{rm\in RM}\sum_{bl\in BL}\theta^v_{rm,bl} \le b^H_t \quad \forall t \qquad (33)
\end{aligned}
$$

$$
z^v_{bl} + \theta^v_{rm,bl} \ge \sum_{t\in T} y_{bl,t}\hat{v}_{rm,bl} \quad \forall rm, bl \qquad (34)
$$

$$
z^v_{bl}, \theta^v_{rm,bl} \ge 0 \quad \forall rm, bl \qquad (35)
$$

Now, by proposing the explained constraints, it is possible to develop the R-HP model such as follows:

$$
Min\ Z_{R-HP} = \pi_1 + \pi_2 + \sum_{rm\in RM}\sum_{bl\in BL}\sum_{t\in T} HH_{rm,bl,t} \qquad (36)
$$

Subject to: constraints(8), (9), (10), (11), (12), (14), (19), (20), (21), (22), (27), (28), (29), (30), (31), (33), (34), (35) (37)

4 Numerical results

In this section, we test the proposed robust optimization model through a set of realistic-size instances from a real lumber supply chain in Canada. In the latter test problems, we consider random supply. The purpose of the numerical experiments is to analyze the trade-off between the level and the cost of robustness. More precisely, we are interested to investigate how increasing the degree of robustness (budget of uncertainty) affects the feasibility and optimality of the nominal (deterministic) solution.

The robust model in this paper is coded in C++ using CPLEX concert technology on a Core i7 CPU 3.40GHz computer with 8.00 GB RAM.

4.1 Results for uncertainty in supply parameters

Uncertain supply affects several terms in the objective function of R-HP model such as harvesting cost and stumpage fee. Moreover, this parameter affects constraint (13) related to the harvesting capacity. In other words, uncertain supply affect on the feasibility and optimality of the R-HP model's solution.

We define γ as the level of variability of the uncertain supply quantities comparing to its nominal value and consider $\gamma = 5\%$ and 20%, so that the scaled deviation of raw material availability in each block, i.e., $\hat{v}_{rm,bl}$, is equal to $\gamma\bar{v}_{rm,bl}$. In order to analyze the effect of uncertain supply in the R-HP model, we assume that $\Gamma^{\pi_1}_{bl}$ and Γ^v_{bl} vary from 0 to $|RM| = 14$ (the worst case). Also, Γ^{π_2} vary from 0 to $|RM||BL| = 700$ since $\Gamma^{\pi_2} = \sum_{bl\in BL}\Gamma^{\pi_1}_{bl}$. The trade-off between robustness (budget of uncertainty) and the cost of robustness is estimated by calculating the cost deviation $(Z^R_{R-HP} - Z^N_{R-HP})/Z^N_{R-HP}$, where Z^R_{R-HP} and Z^N_{R-HP} are the robust and nominal optimal values, respectively.

Another important issue is the analysis of the robust solution in terms of feasibility. As explained earlier, when the budget of uncertainty achieves its maximum value, the robust solutions is feasible. In contrary, the feasibility condition cannot be guaranteed by considering smaller values for the budget of uncertainty. Recall from Section 2 that it is possible to provide probability bounds for the constraint violation in such cases. Based on equation (5), the probabilistic bounds of constraint violation depend only on the number of coefficients subject to uncertainty ($|J|$) and the budget of uncertainty.

Figure (1) represents the percentage increase in the objective function value versus the nominal one for two levels of supply variability for different values of budget of uncertainty in constraints (13) and (23). As expected, when robustness is enforced to the model, the cost is increased in order to envisage worst case scenarios for a certain number of log availability parameters in the objective function (constraints (23)-(24)) and in constraint (13).

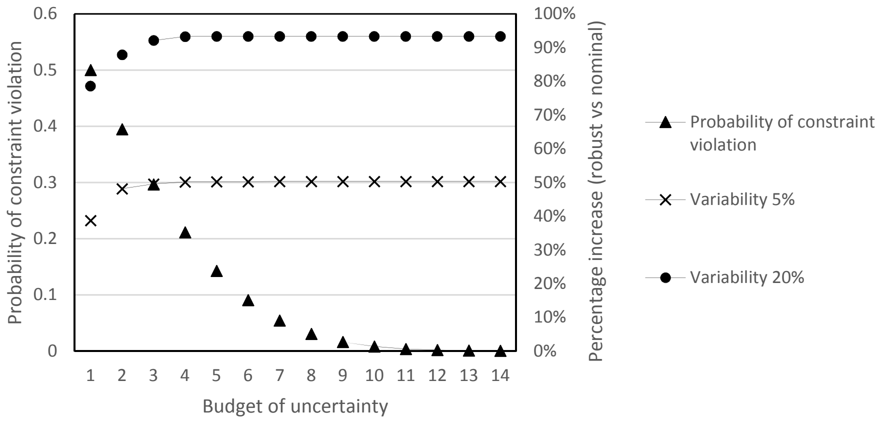

Figure 1: Robust optimal values vs nominal and the probability bounds for constraint violation for random supply

The latter increase in the robust objective function is the effect of such worst case supply scenarios on the harvesting cost and stumpage fee. It is worth noted that such worst case scenarios correspond to the case where the above mentioned log availability parameters ($\hat{v}_{rm,bl}$) take their highest value. In such cases , more logs are harvested and consequently the corresponding harvesting and stumpage costs are increased. Additionally, we can conclude from Figure (1), when the variability level of uncertain parameters is small, the impact of imposing robustness on the objective function is less significant in comparison with higher level of variability. Moreover, the objective function is increased by increasing the budget of uncertainty. When the budget of uncertainty is increased, it indicates that the number of uncertain parameters that take their worst case scenario are increased which in turn, is expected to increase the cost.

As it is observable in Figure (1), the violation probability decreases and tends to reach zero as we increase the budget of uncertainty to its maximum value. This probability is near zero when the budget of uncertainty is greater than 8. In other words, the probability violation in this figure is stable in near 50% of budget of uncertainty. By increasing the budget of uncertainty, the number of uncertain parameters that take their worst-case scenario in the constraint (13) is increased. Thus, the R-HP model tries to find a feasible solution to satisfy the harvesting capacity in constraint (13) for such worst-case scenarios. Consequently, the violation probability of constraint (13) is reduced. Recall from Section 3 that constraint (13) corresponds to the harvesting capacity and might be violated when the majority of log availability parameters ($\hat{v}_{rm,bl}$) take their highest value in the given uncertainty interval.

5 Conclusions

In this paper, we proposed a robust harvesting planning model when log supply and demand are uncertain parameters. We also developed a robust optimization approach that provides some insights into the adjustment of the level of robustness of the solution and protection against uncertainty. the robust model provides an optimal plan which can guarantee against unpredictable log demand and supply and reduce the risk. Moreover, in order to survive in facing with different perturbations and obtain robust decisions in the presence of future uncertainties, it is important to consider the mentioned uncertainties and propose a robust plan. It is not desirable in a management context that the harvesting plan changes significantly while facing with future uncertainties. Furthermore, the main advantage of the proposed robust optimization approach is that it allows studying the trade-off between robustness requirements and the lost in optimality.

We are currently conducting a more comprehensive analysis on the proposed robust optimization model on a set of realistic-size instances from a real lumber supply chain in Canada.

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