A column generation approach for vehicle routing problem in disaster area

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***Résumé* - Cet article considère les opérations de distribution d'aide humanitaire en région sinistrée. Le problème correspond à un problème de tournées de véhicules riche où l'objectif est de minimiser le temps d'accès au dernier sinistré. Nous développons une approche de génération de colonnes afin de résoudre le modèle mathématique proposé. Cette approche repose sur un sous-problème spécifique qui génère des routes avec des chargements prédéterminés à ajouter au problème de maître. Ces routes définissent la séquence de points de la demande à visiter et les quantités des différents produits à livrer.**

***Abstract* - This paper deals with the response phase of humanitarian relief. We model the first operations of distribution of humanitarian aid in disaster area that correspond to a rich vehicle routing problem where we aim at minimizing maximum delivery time. Regarding the model attributes' complexity, we develop a column generation approach to solve the optimization model. This approach relies on a specific pricing problem that generates routes with negative reduced costs to add to the master problem. These routes define the sequence of demand points to visit and the quantities to be carried to each one.**

***Mots clés* – génération de colonnes, logistique humanitaire, problème de tournées de véhicules, livraison partagée.**
***Keywords* – column generation, humanitarian logistics, vehicle routing problem, split delivery.**

1 INTRODUCTION

Natural disasters generate losses and damages that may affect heavily affected countries' economy. It becomes mandatory, especially for countries with a large number of periodic disasters, to take good measures to efficiently face such crisis situations. This paper addresses the logistic aspect of the response phase in natural disasters. It precisely deals with the distribution of humanitarian aids in disaster areas in the first hours after a disaster.

[Haddow et al., 2007] describe the four phases of emergency logistics: "mitigation" and "preparedness", which are pre-crisis phases, "response" and "recovery", which are post-crisis phases. The first two phases are preventive and aim to provide an action plan in order to mitigate the potential disaster damages. Other phases come after the disaster and are intended for the execution of the action plan to limit the disaster damages and to support victims with effective humanitarian aid. After the dangerous critical period, the recovery phase aims to restore the affected infrastructures to return to a standard of living, as it was before the disaster.

In this paper, we focus on the distribution of humanitarian aid during the response phase. We aim to define routes that start from the available humanitarian aid distribution centers (HADC), which are temporary depots, to visit the different affected zones and provide the victims (clients) with their first needs. Given the emergency context, we have to propose effective methods that enable routes to satisfy the victims' needs within a short time.

Our problem is a variant of the vehicle routing problem (VRP) with a mix of complex attributes. First, multiple depots are used to serve the clients (the affected zones) and a client can be visited by multiple routes allowing thus split delivery. Second, the demand of a client is for different types of products. Third, each HADC has a limited capacity in terms of products supply and the number of available vehicles. The fleet available at each HADC is heterogeneous. When determining vehicle routes, the loading and unloading times of vehicles must be taken into account. These times depend on the HADC, the vehicle type and the product type. This former attribute comes from the emergency context when different installations could be used as temporary depots (school, tent, etc...) and are not naturally equipped to serve as depots.

Given the plurality of these attributes, we propose a Mixed Integer Programming model inspired form the MIPs elaborated by [Berkoune et al., 2011] and [Desaulniers, 2010] in order to provide the routing schedule that satisfies all the clients' demands within the shortest delivery time. In other word, we aim to minimize the maximum delivery time of a client. Although this objective may be criticized in classic VRPs encountered in business logistics contexts, we believe it is appropriate in emergency contexts in the first hours after a disaster: it is important to serve all victims as fairly as possible.

Solving this model using exact methods is difficult mainly for large instances, therefore our main contribution is to solve it using a column generation based approach. A feasible solution is first generated using a heuristic approach. The linear relaxation of the proposed MIP is solved by column generation where the pricing sub-problem is an Elementary Shortest Path Model with Resource Constraints (ESPPRC). The ESPPRC is solved using dynamic programming. The MIP with the optimal LP columns is then solved using the Branch-and-Bound procedure of a commercial solver (Cplex 12.5 in our case).

The paper is organised as fallows. Section 2 is a literature review on transportation and distribution models in the emergency context. Section 3 describes the problem and introduces the notation to be used throughout. The MIP model is presented in Section 4. Section 5 describes in details the sub-problem and the dynamic programming approach proposed to solve it. Section 6 reports our preliminary results. Section 7 summarizes the main contributions and presents further research.

2 LITERATURE REVIEW

Several papers addressed distribution problems in disaster areas ([Ozdamar et al., 2004], [Sheu, 2007], [Yi and Kumar, 2007] [Tzeng et al., 2007] , [Balcik et al., 2008], [Berkoune et al., 2011], [Berkoune et al. , 2012]).

[Barbarosoglu and Arda, 2004] developed a two-stage stochastic model with recourse to plan humanitarian aid distribution under demand and supply uncertainty. Their study expands the multimodal multi-commodity deterministic problem of [Haghani and Oh, 1996]. The authors test their model with real data from the earthquake that hit Turkey in 1999.

[Ozdamar et al., 2004] consider a multi-commodity distribution problem with multiple depots. They formulate a mathematical model to determine the delivery routes and the quantities of products to be loaded for each route with the aim of minimizing the unmet demand. The proposed formulation allows to regenerate plans based on changing demand, supply and fleet size. A heuristic approach based on Lagrangian relaxation is proposed to solve the problem and its performance tested with data from the earthquake that hit Turkey in 1999.

[Sheu 2007] used a hybrid optimization approach based on a sequence of steps that rely on demand forecasting and aggregation techniques for affected areas. He assigns a priority to each affected area and uses a multi-objective mathematical model that ensures the demands delivery with respect to supply and vehicles capacities. The model is applied to data from an earthquake that occurred in Taiwan.

[Yi and Kumar, 2007] proposed a linear mixed multi-commodity model, and tested it with data issued from a violent earthquake in Turkey. An ant colony metaheuristic is proposed to optimize the food delivery and people evacuation simultaneously. This method divides the transport problem into two parts: the empty route construction then the loading operation.

[Tzeng et al., 2007] considered a three-objective distribution problem. The first objective minimizes the total cost, the second objective minimizes the travelling time and the third objective maximizes demands satisfaction. A multi-period model is proposed in which most of the parameters and variables are time-dependent. The model is tested with data obtained during an earthquake in Asia in 1999.

[Balcik et al., 2008] consider the humanitarian aid distribution problem in a network containing a single depot with heterogeneous fleet. The objective is to carry available relief supplies to demand points by determining delivery plans for each vehicle considering a planning horizon of several days. The authors propose a solution approach in two stages. In the first stage, the possible delivery routes for each vehicle are generated. Loads decisions are determined in the second stage. The objective is to minimize the sum of transportation costs and penalties due to unmet demand.

[Berkoune et al., 2011] formulate the humanitarian aid distribution problem as MIP where the objective is to minimize the maximum delivery time. Two solution methods are proposed: an exact method and a heuristic method. The exact method exhaustively enumerates all possible routes and uses the branch-and-bound procedure of a commercial solver (CPLEX) to solve the resulting exhaustive model. The heuristic method rather considers a restricted set of routes selected with regard to some criteria. The results show that the heuristic method performs better than the exact approach for large instances in terms of computation time, but offer solutions with unpredictable optimality gaps.

As will be formally explained in Section 3, the problem we address in this paper is a complex multi-commodity multi-depot VRP with split delivery and with a min-max objective. The split delivery assumption is in fact the attribute that makes the problem much more complex. The rest of this section reports the main recent works on split delivery VRP (SDVRP).

SDVRP was introduced by [Dror and Trudeau, 1989] and [Dror and Trudeau, 1990]. They studied properties of optimal solutions and proposed a local search heuristic to solve SDVRP. After that, several heuristics were developed: [Frizzell and Giffin, 1992] designed three greedy heuristics, [Sierksma and Tijssen, 1998] and [Jin et al., 2008] developed two branch-and-price heuristics and a hybrid tabu search algorithm was proposed by [Archetti et al., 2008]. The interest on exact methods has grown up recently especially with the use of branch and price techniques. Recently, [Archetti et al., 2011] and [Desaulniers, 2010] proposed a branch-and-price-and-cut algorithm. Each column generated by the sub-problem represents a route with the quantities to be delivered to each client. The master problem aims at finding the optimal set of routes generated by the sub-problem. The sub-problem is solved using dynamic programming through a labeling algorithm. The master problem is strengthened by a set of valid inequalities to improve the lower bound. These former papers develop some techniques to enhance the set labeling algorithm since the labels' number can be huge for big instances. They also develop an interesting way to generate routes by the pricing problem by considering extreme delivery patterns only. An extreme delivery pattern is a path where at most one demand can be split. Then the master problem finds the optimal routes by making convex combinations of extreme delivery patterns. All these proposed solution approaches do not consider multiple depots, multi-commodity, limited heterogeneous fleet and the loading and unloading times. [Salani et al., 2011] studied a multi-commodity SDVRP called discrete split delivery vehicle routing problem (DSDVRP). This problem assumes that the client's demand is composed by several types of items and the demand can be split only into predetermined discrete orders. The only paper that considers simultaneously split delivery, heterogeneous demand, loading and unloading times and a min-max objective function is proposed by [Yakici and Karasakal., 2013]. They adopted a heuristic approach to solve the model that consists of applying a split operator to the route with the longest travel time. The split operator inserts a new split delivery to a short route in order to decrease the quantity carried to this client by the longest route.

3 PROBLEM DESCRIPTION

In emergency situations, humanitarian aid may consist of tangible products (food, medicine, blankets, water, etc. ...) or services (infrastructures maintenance, restoration of power lines etc ...). These products and services are available in limited quantities in depots that we call the Humanitarian Aid Distribution Centers (HADC). We assume that the quantity and location of effective relief are fixed and do not change during the planning horizon. The planning horizon may extend over a few hours to a full day.

Thereafter, the set of HADC placed near the affected region is denoted L. When a disaster occurs, every home or building in the affected area may require emergency services or tangible humanitarian aid, and thus becomes a point of potential demand. In general, these demand points are grouped according to their geographical coordinates into aggregated demand points. This aggregation technique avoids handling a huge amount of information and then facilitates modeling and solving the problem. In what follows, we use the term "client" to refer to an aggregated demand point. The set of clients in the affected area is denoted I. The nature and amount of product or service in need may vary from one client to another depending on damage's type and severity. We assume them to be fixed for each client during a given planning horizon. Thereafter, all the products requested are denoted by set P and the demand of each client i for each product p is denoted d_{ip}.

Humanitarian aid is deployed from the HADC to clients by vehicles in the case of products or service providers in the case of services. Given the diversity of products and services required in an emergency situation, vehicles available in each HADC can be heterogeneous (light vehicles, trailers, tanks etc ...). Considering the heterogeneity of the vehicles fleet and the crisis situation where we aim at reducing delivery operations time, the optimisation model should take into account the vehicles travel times and the loading and unloading time specific to each product type. In what follows, we denote the set of vehicles types available in each HADC l by K_l. Each vehicle type is characterised by capacity in terms of weight and volume, respectively denoted W_k and V_k and a maximum daily work time denoted D_k which implies that the total travel time of the vehicle including loading and unloading times cannot exceed D_k. We assume that each vehicle must return to the HADC from which it departed even if it provides a service. Furthermore, we denote the volume and weight occupied by a unit of product p v_p and w_p respectively. Finally, the loading and unloading time associated with a product p is denoted ρ_p.

The objective of the considered VRP is to identify routes that can satisfy all the clients' demands for all products, taking into account the characteristics of available vehicles in each HADC l and the quantity of product p supplied at each depot, q_{lp}. Unlike classical VRP where the objective is usually to minimize the total length of routes, we consider here an objective specific to emergency situations which aims to serve all demand points within the shortest possible time. Therefore, our model aims to minimize the maximum delivery time, considering travelling time and loading and unloading times. Indeed, in crisis situations, the main objective is to provide effective disaster relief needed in a relatively short time, especially in the first hours after the disaster. A maximum access time noted τ is pre-specified to ensure that each customer can be served by at least one HADC in a time window lower than τ. We assume that the location of HADCs is chosen so that all clients can be reached from at least one HADC with respect to the time window.

Our problem can be defined as rich vehicle routing problem, yet the split delivery aspect will have the most important impact on the problem's modeling and resolution. In fact, the SDVRP can be defined over a graph $G = (V, A)$ with vertex set $V = I \cup L$ and arc set $A = \{(i,j): i \neq j, (i \in I \cup L \; and \; j \in I) or \; (i \in I \; and \; j \in I \cup L\}$. We assume that the travelling times between each pair of vertices t_{ij} satisfy triangle inequality.

4 MATHEMATICAL MODEL

[Berkoune et al., 2011] adopted a two stage approach to model their MIP. In the first stage, they enumerate all feasible empty routes composed of a maximum of four clients. Each route is composed by the sequence of ordered clients to be visited, the depot from where the route begins, and the vehicle type that will ensure the distribution. Feasible empty routes are defined regarding the time constraints of the vehicle and the time window. For each set of clients, a TSP is solved to obtain the optimal sequence of visit.

After that, the MIP considers these empty routes and determines the routes to be chosen and the quantities to be delivered by each route for each product in order to satisfy all the demands in the shortest possible time.

This mathematical model requires the following notations:

- $\sigma_{rl} = 1$, if route r is associated with depot l; 0, otherwise

- $\varphi_{ri} = 1$, if route r visit client i; 0, otherwise.
- $\alpha_{rlk} = 1$, if route r starts from HACD l and is associated with vehicle type k; 0, otherwise
- $\alpha'_{rk} = 1$, if route r is associated with vehicle type k; 0, otherwise
- T_r^d= travelling time of route r with returning to the depot and excluding loading and unloading times.
- $\tilde{T}_r^d$= travelling time of route r without returning to the depot and excluding loading and unloading times.

Decision variables

- y_{rji} = quantity of product p delivered to client i by route r
- $x_r = 1$ if route r is chosen, 0 otherwise
- L_{max}= the maximum delivery time.

$$Min\ L_{max} \tag{1}$$

$$\sum_{r\in R} \varphi_{ri}\, y_{rji} = d_{ij} \qquad \forall i \in I, j \in J \tag{2}$$

$$\sum_{r\in R}\sum_{i\in I} \sigma_{rl}\, \varphi_{ri}\, y_{rji} \le p_{jl} \qquad \forall j \in J, l \in L \tag{3}$$

$$\sum_{r\in R}\sum_{i\in I}\sum_{j\in J} \varphi_{ri}\, \alpha'_{rk}\, \omega_j\, y_{rji} \le W_k \qquad \forall k \in K \tag{4}$$

$$\sum_{r\in R}\sum_{i\in I}\sum_{j\in J} \varphi_{ri}\, \alpha'_{rk}\, \vartheta_j\, y_{rji} \le V_k \qquad \forall k \in K \tag{5}$$

$$\sum_{j\in J}\sum_{i\in S} \rho_p\, y_{rji} + T_r^d \le D_k\, x_r \qquad \forall r \in R \tag{6}$$

$$\sum_{j\in J}\sum_{i\in S} \rho_p\, y_{rji} + \tilde{T}_r^d \le L_{max} \qquad \forall r \in R \tag{7}$$

$$\sum_{r\in R} \alpha_{rlk}\, x_r \le M_{lk} \qquad \forall l \in L, \forall k \in K \tag{8}$$

$$L_{max} \ge 0\ ;\ x_r \in \{0,1\}, y_{rji} \ge 0 \qquad \forall r \in R,\ j \in J, i \in I \tag{9}$$

Objective function (1) aims at minimizing the maximum delivery time. Constraints (2) ensure demand satisfaction of each client for each product. Constraints (3) guarantee that the quantity of product p delivered from each depot l does not exceed the depot supply. Constraints (4), (5) and (6) ensure the respect of vehicle's type specification (weight, volume and maximum service time). Constraints (7) set L_{max} as an upper bound for every route's delivery time. Constraints (8) ensure that routes starting from each depot respect the available vehicle fleet.

Given the huge number of integer variables (feasible routes), we propose to solve this model using a column generation based approach. Typically, this would imply solving the LP relaxation of model (1)-(9) with column generation. In this case, only a subset of routes is considered in the master problem in the first iteration. Promising routes are added progressively by solving a pricing sub-problem until an optimal LP solution is identified. However, model (1)-(9) is clearly not appropriate for column generation since the number of constraints depend on the number of variables. If kept as it is, this would imply a simultaneous dynamic generation of variables and constraints. To handle this, we propose to use another LP formulation inspired by the work of [Desaulniers, 2010]. Observe that given the objective function of minimizing the maximum travel time with no fixed costs associated with vehicles use, an optimal LP solution would use all the vehicles available at all the depots. Moreover, the pricing sub-problem will define the shortest route with the quantities to be carried. To moderate the number of generated routes, we will consider as in [Archetti et al., 2011] the generation of extreme delivery patterns. An extreme delivery pattern is a path composed by at most one split delivery, full deliveries and unit deliveries. The initial solution is obtained using a heuristic approach that will be described later.

The master problem requires the following notations:

- W_r: set of all extreme delivery patterns compatible with route r.
- $V_{l,k}$: set of all vehicles ε of type k available at HADC l.
- I_r : set of clients visited by the route r
- δ_{iwp} : quantity of product p delivered to client i in pattern w
- $\alpha_{rlk} = 1$, if route r starts from HADC l and is associated with vehicle type k; 0, otherwise
- $\tilde{T}_r^d$= travelling time of route r without returning to the depot and excluding loading and unloading times.

Decision variables

- $\theta_{rw\varepsilon} > 0$ if extreme pattern w is chosen to form the route r assigned to vehicle ε , 0 otherwise.
- $\theta_{r\varepsilon} = 1$ if route r is chosen for vehicle ε , 0 otherwise
- L_{max}= the maximum delivery time.

$$Min\ L_{max} \tag{10}$$

$$\sum_{l\in L}\sum_{k\in K_l}\sum_{r\in R}\sum_{w\in W_r}\sum_{\varepsilon\in V_{l,k}} \alpha_{rlk}\delta_{iwp}\theta_{rw\varepsilon} \ge d_{ip} \qquad \forall i \in I, \forall p \in P \tag{11}$$

$$\sum_{i\in I}\sum_{k\in K_l}\sum_{r\in R}\sum_{w\in W_r}\sum_{\varepsilon\in V_{l,k}} \alpha_{rlk}\, \delta_{iwp}\theta_{rw\varepsilon} \le q_{lp} \qquad \forall l \in L, \forall p \in P \tag{12}$$

$$\sum_{r\in R}\sum_{w\in W_r} (\tilde{T}_r^d + \sum_{p}\sum_{i'\in I_r} \rho_p \delta_{i'wrp})\, \theta_{rw\varepsilon} \le L_{max} \qquad \forall \varepsilon \in V_{l,k} \tag{13}$$

$$\sum_{r\in R}\sum_{w\in W_r}\sum_{\varepsilon\in V_{l,k}} \alpha_{rlk}\theta_{rw\varepsilon} \le M_{lk} \qquad \forall l \in L, \forall k \in K \tag{14}$$

$$\theta_{r\varepsilon} = \sum_{w\in W_r} \theta_{rw\varepsilon} \qquad \forall r \in R \tag{15}$$

$$L_{max} \ge 0\ ; \theta_{rw\varepsilon} \ge 0\ ;\ \theta_{r\varepsilon} \in \{0,1\} \qquad \forall r \in R, \forall w \in W_r \tag{16}$$

Objective function (10) remains the same as (1). Constraints (11), (12), (13), and (14) replace respectively (2), (3), (7), and (8). Constraints (4), (5) and (6) are implicitly respected during the subproblem resolution. Constraints (15) ensure that every $\theta_{r\varepsilon}$ is a result of a convex combination of the variables $\theta_{rw\varepsilon}$. We consider $\theta_{rw\varepsilon}$ as continuous variables so we enable finale routes $\theta_{r\varepsilon}$ to have more than one split. The remaining constraints are integrality and sign constraints.

5 Column Generation Approach

As the master problem (MP) resolution with branch and bound method is complex due to the huge number of integer variables, we propose to solve it using a column generation approach. We generate routes $\theta_{rw\varepsilon}$ with negative reduced costs to the linear relaxation of the MP (LRMP) using the sub-problem. Once we cannot add more routes to the linear LRMP, we solve the MP using Branch and Bound method. Note that the LRMP is initially solved with a restricted set of routes generated using a heuristic approach.

5.1 Heuristic Approach for initial solution

The heuristic we developed for producing an initial solution provides the set of routes to consider including the sequence of clients to visit, the vehicle that will ensure the transport for each route and the quantities of the different products needed to be loaded in each vehicle. After executing all the instructions of the program, all demand points have to be satisfied while minimizing the maximum delivery time.

The algorithm that describes the different stages of the heuristic begins by identifying the clients who have to be served by each centre depending on the maximum access time that has to be computed before. Then, for each centre, we generate all the possible combinations of clients. The sets of combinations are used after that to generate the sequences by permuting the elements of each set. After computing the initial value of L_{max}, we identify for each centre the empty routes whose the duration is lower than L_{max}. Then we select the vehicle that will ensure the transportation on this route which must have the smallest fixed loading time. The next step consists of calculating the proportions between all clients' demands for each product and the vehicle capacity in order to allow a specified capacity to each product type depending on these proportions. After that, we calculate the demands' proportions of each product among all the route clients to determine the quantity to be carried to each one that respects the vehicle's capacity and the centre's supply. This loading procedure ensures a fair service between all the clients. After that we update the vehicles' availability, the clients' demands and the centers' offers. Next we eliminate the sequences that contain the clients whose demand are totally satisfied. Then we verify if all the clients' demands are satisfied the program ends, else we increase $L_{max}$and we repeat the process until satisfying all the demands.

5.2 Sub-problem formulation

In a column generation approach, the sub-problem has to find the route with the least reduced cost for each HADC and each vehicle available at this depot. In our case, we consider the sub-problem as an elementary shortest path problem with resource constraints ESPPRC. The sub-problem defines the route r and the extreme delivery pattern w associated with $\theta_{rw\varepsilon}$. We consider the following dual variables:

- π_{ip} : non-negative dual variable of constraint (11)
- μ_{lp} : non-positive dual variable of constraint (12)
- λ_{ε} : non-positive dual variable of constraint (13)
- z_{lk} : non-positive dual variable of constraint (14)

As we consider a sub-problem for each depot l and for each vehicle type k available in the depot, the reduced cost can be formulated as follows:

$$\overline{c_{rw}} = \tilde{T}_r^d \lambda_\varepsilon - \sum_{i,p} \delta_{ip}\, \pi_{ip} + (\sum_{i,p} \rho_p \delta_{ip})\lambda_\varepsilon + z + \sum_{i,p} \delta_{ip}\mu_p$$

We define for each sub-problem depending on depot l and vehicle type k the graph $G = (V, A)$ with vertex set $V = N_l \cup l$ (N_l is the set of clients that can be served by depot l) and arc set $A = \{(i,j): i \neq j, (i \in N_l \cup l \; and \; j \in I) or (i \in N_l \; and\; j \in I \cup l\}$. Let xij=1 if arc ij belongs to the extreme delivery pattern $\theta_{rw\varepsilon}$, 0 otherwise.

The reduced cost can then be expressed as following:

$$\overline{c_{rw}} = (\sum_{i,j} c_{ij})\lambda_\varepsilon - \sum_{i,p} \delta_{ip}\, \pi_{ip} + (\sum_{i,p} \rho_p \delta_{ip})\lambda_\varepsilon + z + \sum_{i,p} \delta_{ip}\mu_p$$

Where

$$c_{ij} = t_{ij}x_{ij} \text{ , if } j \neq l$$

5.3 Solving the subproblem

We propose a label setting algorithm to solve the sub-problem. Our sub-problem is a new variant of ESPPRC since we consider the multi-commodity context and the minimization of the maximum delivery time without returning to the depot that depends on traveling time and loading and unloading times. As we consider a sub-problem for each depot and each vehicle type, we will consider a graph composed by one source node (the depot l), the set of clients $i \in N_l$ that can be reached by this depot with respect to time window τ and finally returning to the depot l. Vertices that represent clients are duplicated as much as we have product types. The set labeling algorithm relies on assigning labels to each partial path starting from l then visiting a sequence of clients and finally returning to depot l. A label E is associated with a vertex (i, p) and a partial path is defined as follows: $E = (C, S, L_p, V^1, \dots, V^n, F, \Delta)$ where:

- C is the path's reduced cost
- S is the time resource consumption that includes travelling time and loading and unloading times
- L is a vector composed by |P| elements that indicates the quantity loaded of each item's type when leaving the vertex (i, p)
- V^j indicates whether or not client j has been visited or cannot be visited anymore because of the time constraints. If the client is already visited, it indicates his delivery type. $V^j = 1$: full delivery, $V^j = 2$: unit delivery, $V^j = 3$: split delivery, $V^j = 0$: client can be visited, $V^j = -1$: client cannot be visited anymore
- F : indicates whether or not we have a split delivery
- Δ_{jp} : indicates the quantity carried for client j of every product p if its demand is split. We assume that if a partial path contains a split delivery, all the demand types for this client are allowed to be split.

A label E is said to be feasible if: $S \leq \min\{\tau, D\}$, $\sum_p v_p L_p \leq V$, $\sum_p w_p L_p \leq W$, $L_p \leq q_p$, and $F \leq 1$. Like [Ceselli et al., 2009], we distinguish internal and external extensions. Internal extensions correspond to transition between two nodes for the same client but with different items, thus an extension from a node (i, p) to another node (i, p'). External extensions are performed while changing the clients' locations, so from a vertex (i, p) to a vertex (j, p'). Consequently, each path is composed by several extension blocks, each composed by an external extension and a set of internal extensions.

Given a feasible label E_{ip} it can be extended as internal extension up to three times along an arc$((i,p),(i,p'))$ to form a label $E_{ip'}$ which components are computed as follows:

Case 1: if $V^i = 1$ then: $S_{ip'} = S_{ip} + \rho_{p'} d_{ip'}$, $L_{p'} = L_{p'} + d_{ip'}$, $F_{ip'} = F_{ip}$, $\Delta_{ip'} = \Delta_{ip}$, $C_{ip'} = C_{ip} + \overline{c_{ip}} - c_{(i,p),(i,p')} - z$

Case 2: if $V^i = 2$ then: $S_{ip'} = S_{ip} + \rho_{p'}$, $L_{p'} = L_{p'} + 1$, $F_{ip'} = F_{ip}$, $\Delta_{ip'} = \Delta_{ip}$, $C_{ip'} = C_{ip} + \overline{c_{ip'}} - c_{(i,p),(i,p')} - z$

Case 3: if $V^i = 3$ then: $S_{ip'} = S_{ip} + \rho_{p'}\ \Delta_{ip'}$, $L_{p'} = L_{p'} + \Delta_{ip'}$, $F_{ip'} = 1$, $\Delta_{ip'} = min\{d_{ip'},\ q_{p'},\ \frac{V}{v} - L_{p'},\ \frac{W}{w} - L_{p'},\ \frac{\tau - S_{ip}}{\rho_{p'}},\ \frac{D - S_{ip}}{\rho_{p'}}\}$, $C_{ip'} = C_{ip} + \overline{c_{ip'}} - c_{(i,p),(i,p')} - z$

These three forms of internal extensions respect the fact that at most one demand's client can be split for all the items types. For the other path's clients, we assume that if we provide a client with its full demand (resp. one unit's demand) for an item type, we proceed in the same way for all the other items' types. To perform these internal extensions, we have to respect some rules while performing the external extensions to guarantee the extension's feasibility. To extend a label from a vertex (i,p) to a vertex (j,p'), also 3 cases can occur:

Case 1: if $V^j = 0$, $\sum_p v_p(L_p + d_{jp}) \le V$, $\sum_p w_p(L_p + d_{jp}) \le W, \forall p \in P\ L_p + d_{jp} \le q_p$, $S_{ip} + t_{ij} + \sum_{p'} \rho_{p'} d_{jp'} \le min\{\tau, D\}$, then a full delivery occurs and the new label's components are calculated as follows : $S_{jp'} = S_{ip} + t_{ij} + \rho_{p'} d_{ip'}$, $L_{p'} = L_{p'} + d_{jp'}$, $F_{jp'} = F_{ip}$, $\Delta_{jp'} = \Delta_{ip}$, $C_{jp'} = C_{ip} + \overline{c_{jp'}}$, $V^j = 1$

Case 2: if $V^j = 0$, $\sum_p v_p(L_p + 1) \le V$, $\sum_p w_p(L_p + 1) \le W$, $\forall p \in P\ L_p + d_{jp} \le q_p - 1$, $S_{ip} + t_{ij} + \sum_{p'} \rho_{p'} \le min\{\tau, D\}$, then a delivery of one unit occurs and the new label's components are calculated as follows : $S_{jp'} = S_{ip} + t_{ij} + \rho_{p'}$, $L_{p'} = L_p + 1$, $F_{jp'} = F_{ip}$, $\Delta_{jp'} = \Delta_{ip}$, $C_{jp'} = C_{ip} + \overline{c_{jp'}}$, $V^j = 2$

Case 3: if $V^j = 0$, $S_{ip} + t_{ij} \le min\{\tau, D\}$, $F_{ip} = 0$ then a split delivery occurs and the new label's components are calculated as follows : $S_{jp'} = S_{ip} + t_{ij} + \rho_{p'} \Delta_{jp'}$, $L_{p'} = L_{p'} + \Delta_{jp'}$, $F_{jp'} = 1$, $\Delta_{jp'} = min\{d_{ip'},\ q_{p'}, \frac{V}{v} - L_{p'}, \frac{W}{w} - L_{p'},\ \frac{\tau - S}{\rho_{p'}},\ \frac{D - S}{\rho_{p'}}\}$, $C_{jp'} = C_{ip} + \overline{c_{jp'}}$, $V^j = 3$

As the labels' number can become huge, we adopt a dominance rule to discard less effective paths. Regarding a vertex (i,p) and 2 labels associated with this vertex E_1 and E_2, label E_1 is said to be dominated by E_2 if $C_2 \le C_1$, $S_2 \le S_1$, $(L_2)_p \le (L_1)_p$ and $F_2 \le F_1$. To find the shortest path, we propose to extend the feasible labels using the extension functions, and then we apply the dominance rule at each node to discard the dominated ones. After considering all the labels, we choose at the ending depot l the labels that have negative reduced costs. These labels are then added to the master problem.

A high-level description of the set labelling algorithm is described by algorithm 1.

Algorithm 1
Repeat until finding a negative path
  Define vertices (i,p) that can be served by depot l within τ
  Create an initial label E_0
  Set $UL_0 := E_0$ and $TL_0 := \emptyset$
  For $(i,p) \in N \backslash \{l\}$ do
    Set $UL_{(i,p)} := \emptyset$ and $TL_{(i,p)} := \emptyset$
  While $\bigcup_{(i,p) \in N} UL_{(i,p)} \ne \emptyset$
    Choose a label $E_{(i,p)} \in UL_{(i,p)}$ $(UL_{(i,p)} \ne \emptyset)$
    For all $((i,p),(j,p')) \in A$ do
      Extend $E_{(i,p)}$ over arc $((i,p),(j,p'))$ to create a new label $E_{(j,p')}$ using extension functions
    If $E_{(j,p')}$ is feasible then
      Set $UL_{(j,p')} = UL_{(j,p')} \cup E_{(j,p')}$
      Discard dominated labels from $UL_{(j,p')} \cup TL_{(j,p')}$
    Set $UL_{(i,p)} := UL_{(i,p)} \backslash \{E_{(i,p)}\}$
    Set $TL_{(i,p)} := TL_{(i,p)} \cup \{E_{(i,p)}\}$
    The label associated with the vertex l with the negative reduced costs are added to the master problem

6 PRELIMENARY RESULTS

In order to test our approach we are using academic instances. All tests were performed by a PC Intel Core 2 Duo, 3 GHz, and 4Go RAM. CPLEX 12.5 is used to solve the mathematical programs. The column generation approach is coded with C++ on Microsoft Visual Studio environment.

6.1 Instances

We are using 3 classes of instances denoted from $C1$ to $C3$. Each class is characterised by the number of depots (L) and the number of demand points (I). These characteristics are given in table 1.

Table1. Instances Classes

Class	*L*	*I*
$C1$	3	15
$C2$	4	15
$C3$	4	20

For all instance classes, we consider two vehicle types, two available vehicles of each type in each depot and two product types. For each class we consider several instances where we vary the geographic locations of both depots and demand points and the quantity of both available supplies and demands.

6.2 Numeric results

Solution results of our column generation based approach are summarized in tables 2 and 3 for 5 instances. We also solved the model $(1) - (9)$ using classic Branch&Bound by CPLEX in order to evaluate our approach's efficiency. We then compare our method to CPLEX taking into account the solution quality and the execution time.
Table 2 displays the objective function values $(Lmax)$ for each instance. Column A shows our column generation approach results and column B shows CPLEX results. We then show the gap (in percentage) between the solution output by our approach and the exact method. $(gap = (A - B)/B)$.

Table 2. Solution quality

Class	Instance	*A*	B	Gap
$C1$	1	65.99	66.39	-0.6%
	2	78.42	73.46	6.75%
$C2$	3	65.33	61.3	6.57%
$C3$	4	67.15	61.67	8.88%
	5	78.4	71.12	10.23%

We notice that the gap is proportional to the instance size and its average value is around 6%. We notice also that for the first instance our method provides a better solution than CPLEX. We recall that with CPLEX, we are enumerating all possible routes that contain up to 4 clients, however there is no such restriction for our column generation approach (so a solution can contain a route with more than 4 clients).

We display in table 3 the execution times (in seconds) of our approach (column A) and the exact method (column B).

Table 3. Solution time

Class	Instance	*A*(s)	B(s)
$C1$	1	41.49	249.09
	2	101.75	7.71
$C2$	3	51.92	8.6
$C3$	4	171.86	543.10
	5	122.83	3940

We notice that the average execution time of our approach is around 100 seconds for all instance classes. However the execution time with CPLEX varies noticeably depending on the instance size. Note that the number of feasible routes grows exponentially if we enlarge the instance size. Our results show that CPLEX is very efficient to solve small instances, but its efficiency decreases with bigger instances. The difference of execution times for instances 4 and 5 between our approach and CPLEX shows that our resolution method is effective especially as the gaps are relatively low ($<= 10\%$).

7 CONCLUSION

In this paper, we have introduced a new column generation approach to solve a complex VRP in a disaster area. This method relies on an ESPPRC that generates routes and defines quantities to be delivered to each client for each product. The sub-problem is defined for each depot and for each vehicle type, since we have a multi-sourcing problem with a limited heterogeneous fleet. The routes generated by the sub-problem are called extreme delivery patterns, where only one split is allowed. As we consider a multi-commodity VRP, a split delivery can occur for only one client in an extreme delivery pattern, yet for all the items' types he needs. Since our objective function aims at minimizing the maximum delivery time that depends on both travelling time and loading and unloading times, the decision on quantities to be carried by each route has an important impact on the objective function value. Thus, after generating the extreme delivery routes, the master problem can combine them to allow more than one split delivery by route. To solve the ESPPRC we developed a set labeling algorithm inspired from [Desaulniers, 2010] and [Archetti et al., 2011] that considers the multi-commodity context and the loading and unloading times.

Our preliminary results are promising. In order to improve our solution approach, we are working on accelerating strategies to speed up the set labeling algorithm specially when dealing with large instances. We can use a heuristic approach to generate rapidly paths with negative reduced costs at the early stages. We can also use bounded bidirectional search that was used by [Righini and Salani, 2006] and [Ceselli et al., 2006]. This method generates forward and backward partial paths than links them together with respect to feasibility conditions.

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